1.4
Linear Modeling

- Model a set of data with a linear function.
- Fit a regression line to a set of data; then use the linear model to make predictions.

Mathematical Modeling

When a real-world problem can be described in a mathematical language, we have a mathematical model.

The mathematical model gives results that allow one to predict what will happen in that real-world situation.

Curve Fitting

Curve fitting is the aspect of modeling that tries to find a function that fits observations as well as possible.

One examines scatter plots to determine which kind of function fits the data.

In this section, we will explore linear functions.

Example

Model the data in the table on the number of U.S. households with cable television with a linear function. Then predict the number of cable subscribers in 2010.
Example (continued)

Choose any two data points to determine the equation. We’ll use (2, 81.5) and (6, 94.0). First, find the slope.

\[ m = \frac{94.0 - 81.5}{6 - 2} = 3.125 \]

Substitute 3.125 for \( m \) and use either point, we’ll use (6, 94.0) in the point-slope equation.

\[ y - 94.0 = 3.125(x - 6) \]
\[ y = 3.125x + 75.25 \]

\( x \) is number of years after 1999, \( y \) is in millions.

Here’s the graph of this line.

Here’s the graph of this line.

Linear Regression

Linear regression is a procedure that can be used to model a set of data using a linear function. We use the data on the number of U.S. households with cable television. We can fit a regression line of the form \( y = mx + b \) to the data using the LINEAR REGRESSION feature on a graphing calculator.

Example

Fit a regression line to the data given in the table. Use the function to predict the number of cable subscribers in 2010.

<table>
<thead>
<tr>
<th>Year, x</th>
<th>Number of Cable Television Subscribers (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999, 0</td>
<td>76.4</td>
</tr>
<tr>
<td>2000, 1</td>
<td>78.6</td>
</tr>
<tr>
<td>2001, 2</td>
<td>81.5</td>
</tr>
<tr>
<td>2002, 3</td>
<td>87.8</td>
</tr>
<tr>
<td>2003, 4</td>
<td>88.4</td>
</tr>
<tr>
<td>2004, 5</td>
<td>92.4</td>
</tr>
<tr>
<td>2005, 6</td>
<td>94.0</td>
</tr>
<tr>
<td>2006, 7</td>
<td>95.0</td>
</tr>
</tbody>
</table>

Solution:
Enter the data in lists on the calculator. The independent variables or \( x \) values are entered into List 1, the dependent variables or \( y \) values into List 2. The calculator can then create a scatterplot.
Example (continued)

Here are screen shots of the calculator showing Lists 1 and 2 and the scatterplot of the data.

Here are screen shots of selecting the LINEAR REGRESSION feature from the STAT CALC menu.

From the screen on the right, we find the linear equation that best models the data is

\[ y = 2.863095238x + 76.7416667. \]

Example (continued)

The calculator can also graph the regression line on the same graph as the scatterplot.

Substitute 11 into the regression equation.

It predicts 108.2 million cable subscribers in 2010.