2.3 Exercise Set

Definition:

The composite function $f \circ g$, the composition of f and g, is defined as

$$(f \circ g)(x) = f(g(x)),$$

where x is in the domain of g and $g(x)$ is in the domain of f.

Find a) $(f \circ g)(x)$ *and* b) $(g \circ f)(x)$ *and the domain of each.*

12. $f(x) = 3x - 2$, $g(x) = x^2 + 5$

a) $(f \circ g)(x) = f(g(x))$

$= f(x^2 + 5)$

$= 3(x^2 + 5) - 2$

$= 3x^2 + 15 - 2 = 3x^2 + 13$

$D_{f \circ g}: x \in (-\infty, \infty)$

b) $(g \circ f)(x) = g(f(x))$

$= g(3x - 2)$

$= (3x - 2)^2 + 5$

$= 9x^2 - 12x + 4 + 5$

$= 9x^2 - 12x + 9$

$D_{g \circ f}: x \in (-\infty, \infty)$

16. $f(x) = \frac{6}{x}$, $g(x) = \frac{1}{2x+1}$

$2x+1 \neq 0$
$2x \neq -1$
$x \neq -\frac{1}{2}$

stop: $x \neq -\frac{1}{2}$

a) $(f \circ g)(x) = f\left(\frac{1}{2x+1}\right)$

$= \frac{2x+1}{2x+1} \cdot \frac{6}{\frac{1}{2x+1}} = \frac{12x+6}{1} = 12x+6$

$D_{f\circ g}: x \in (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

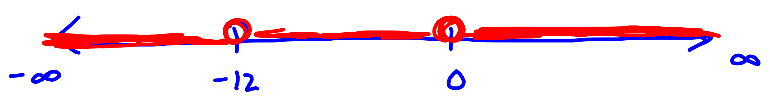
stop: $x \neq 0$

b) $(g \circ f)(x) = g\left(\frac{6}{x}\right) = \frac{1}{2\left(\frac{6}{x}\right)+1}$

$= \frac{x}{x} \cdot \frac{1}{\frac{12}{x}+1} = \frac{x}{12+x}$

stop: $x \neq -12$

$D_{g\circ f}: (-\infty, -12) \cup (-12, 0) \cup (0, \infty)$



25. $f(x) = 3x - 7,\ g(x) = \dfrac{x+7}{3}$

a) $(f \circ g)(x) = f\left(\frac{x+7}{3}\right) = 3\left(\frac{x+7}{3}\right) - 7$

$= x + 7 - 7 = x$ $\quad D_{f\circ g}: x \in (-\infty, \infty)$

b) $(g \circ f)(x) = g(3x-7) = \dfrac{(3x-7)+7}{3}$

$= \frac{3x}{3} = x$ $\quad D_{g\circ f}: x \in (-\infty, \infty)$

28. $f(x) = \sqrt{x},\ g(x) = 2 - 3x$

a) $(f \circ g)(x) = f(2-3x)$

$= \sqrt{2-3x}$

$D_{f\circ g}: \quad 2 - 3x \geq 0$

$-3x \geq -2$

$x \leq \frac{2}{3}$

$x \in \left(-\infty, \frac{2}{3}\right]$

stop: $x \geq 0$

b) $(g \circ f)(x) = g(\sqrt{x})$

$= 2 - 3\sqrt{x}$

domain: $x \geq 0$

$x \in [0, \infty)$

35. $f(x) = \dfrac{1-x}{x}$, $g(x) = \dfrac{1}{1+x}$

a) $(f \circ g)(x) = \dfrac{1 - \left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)}$ stop: $x \neq -1$

$= \dfrac{1+x}{1+x} \cdot \dfrac{1 - \frac{1}{1+x}}{\frac{1}{1+x}}$

$= \dfrac{1+x-1}{1} = \dfrac{x}{1} = x$

domain: $x \in (-\infty, -1) \cup (-1, \infty)$

b) $(g \circ f)(x) = \dfrac{1}{1+\left(\frac{1-x}{x}\right)}$ stop: $x \neq 0$ $= \dfrac{x}{x} \cdot \dfrac{1}{1+\frac{1-x}{x}}$

$= \dfrac{x}{x+(1-x)} = \dfrac{x}{1} = x$

domain: $(-\infty, 0) \cup (0, \infty)$