

Transformations

2.5

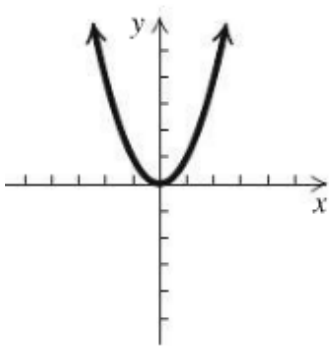
Objectives:

- Describe how a function is obtained from basic graphs.
- Write the equation for a function after various transformations.

The basic functions we will be working with. As we go through the course, we will meet further functions.

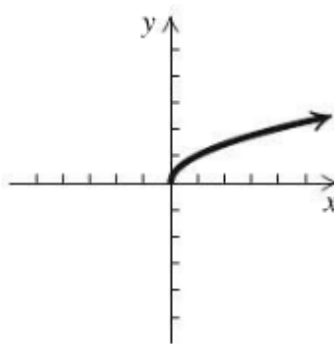
Squaring function:

$$y = x^2$$



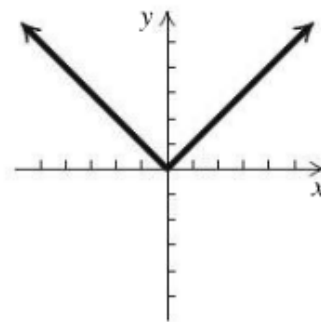
Square root function:

$$y = \sqrt{x}$$



Absolute-value function:

$$y = |x|$$



The building blocks for new functions

VERTICAL TRANSLATION

For $b > 0$:

the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted *up* b units;

the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted *down* b units.

Compare $g(x) = x^2 + 5$ with $h(x) = x^2$

Compare $f(x) = \sqrt{x} - 2$ with $g(x) = \sqrt{x}$

HORIZONTAL TRANSLATION

For $d > 0$:

the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted *to the right* d units;

the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted *to the left* d units.

Compare $f(x) = |x|$ with $g(x) = |x - 2|$

Compare $f(x) = x^2$ with $g(x) = (x + 4)^2$

REFLECTIONS

The graph of $y = -f(x)$ is the **reflection** of the graph of $y = f(x)$ across the x -axis.

The graph of $y = f(-x)$ is the **reflection** of the graph of $y = f(x)$ across the y -axis.

If a point (x, y) is on the graph of $y = f(x)$, then $(x, -y)$ is on the graph of $y = -f(x)$, and $(-x, y)$ is on the graph of $y = f(-x)$.

Compare $f(x) = \sqrt{x}$ with $g(x) = -\sqrt{x}$



Compare $f(x) = \sqrt{x}$ with $g(x) = \sqrt{-x}$

VERTICAL STRETCHING AND SHRINKING

The graph of $y = af(x)$ can be obtained from the graph of $y = f(x)$ by stretching vertically for $a > 1$, or shrinking vertically for $0 < a < 1$.

Compare $f(x) = \sqrt{x}$ with $g(x) = 2\sqrt{x}$ and $h(x) = 0.5\sqrt{x}$

HORIZONTAL STRETCHING AND SHRINKING

The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by shrinking horizontally for $c > 1$, or stretching horizontally for $0 < c < 1$.

Compare $f(x) = |x|$ with $g(x) = |2x|$ and $h(x) = |0.5x|$

Describe how the graph of the function can be obtained from one of the basic graphs on p. 129. Then graph the function by hand or with a graphing calculator.

11. $h(x) = \frac{1}{2}|x| - 2$

12. $g(x) = -|x| + 2$

26. $f(x) = |x - 3| - 4$

Write an equation for a function that has a graph with the given characteristics.

49. The shape of $y = x^2$, but reflected across the x -axis and shifted right 8 units

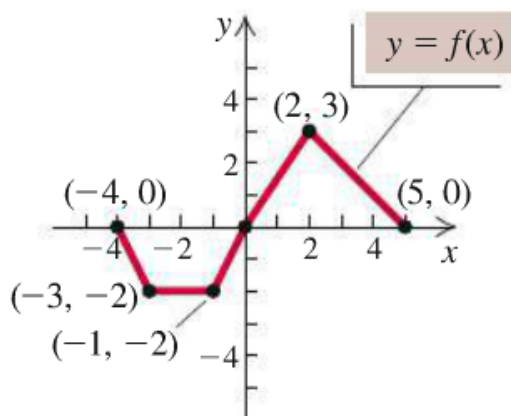
50. The shape of $y = \sqrt{x}$, but shifted left 6 units and down 5 units

51. The shape of $y = |x|$, but shifted left 7 units and up 2 units

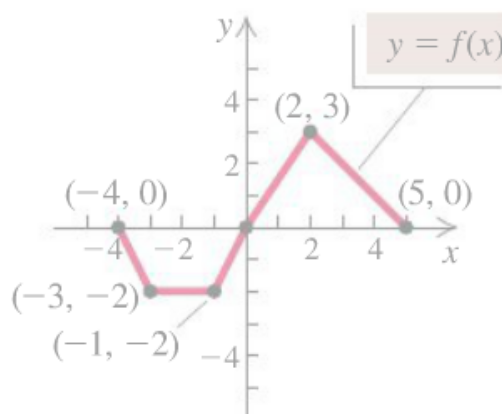
55. The shape of $y = x^2$, but reflected across the x -axis and shifted right 3 units and up 4 units

56. The shape of $y = |x|$, but stretched horizontally by a factor of 2 and shifted down 5 units

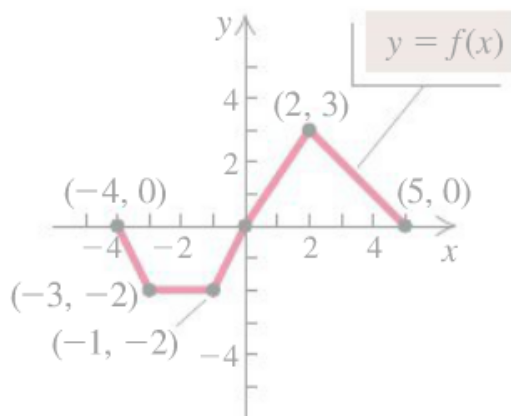
57. The shape of $y = \sqrt{x}$, but reflected across the y -axis and shifted left 2 units and down 1 unit



59. $g(x) = -2f(x)$



65. $g(x) = f(-x)$



66. $g(x) = -f(x)$

