Quadratic Equations, Functions, Zeros, and Models

3.2

QUADRATIC EQUATIONS

A **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0, \quad a \neq 0,$$

where a, b, and c are real numbers.

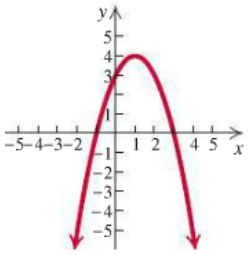
QUADRATIC FUNCTIONS

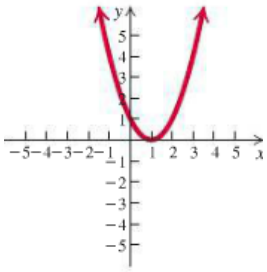
A **quadratic function** f is a function that can be written in the form

$$f(x) = ax^2 + bx + c, \quad a \neq 0,$$

where a, b, and c are real numbers.

In Exercises 21–28, use the given graph to find **(a)** *the x-intercepts and* **(b)** *the zeros of the function.*





The Principle of Zero Products: If $ab = 0$ is true, then $a = 0$ or $b = 0$, and if $a = 0$ or $b = 0$, then $ab = 0$.

$$(5x - 2)(2x + 3) = 0$$

$$x^2 - 8x - 20 = 0$$

The Principle of Square Roots: If $x^2 = k$, then $x = \sqrt{k}$ *or* $x = -\sqrt{k}$.

$$4x^2 - 12 = 0$$

$$4x^2 + 12 = 0$$

$3y^3 - 5y^2 - 2y = 0$

$3x^3 + x^2 - 12x - 4 = 0$

Solve graphically. Round solutions to three decimal places, where appropriate.

$3x^2 + 5x = 3$

Perfect-Square Trinomials

$x^2 + 12x + 36$

$x^2 - 10x + 25$

$x^2 + kx + \_\_\_\_\_$

Completing the Square

$x^2 + 6x + 13 = 0$

$2x^2 - 5x - 3 = 0$

Solve for x

$$ax^2 + bx + c = 0, a \neq 0$$

Quadratic Formula

THE QUADRATIC FORMULA

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The Discriminant

DISCRIMINANT

For $ax^2 + bx + c = 0$, where a, b, and c are real numbers, $a \neq 0$:

$b^2 - 4ac = 0 \longrightarrow$ One real-number solution;

$b^2 - 4ac > 0 \longrightarrow$ Two different real-number solutions;

$b^2 - 4ac < 0 \longrightarrow$ Two different imaginary-number solutions, complex conjugates.

For each of the following, find the discriminant, $b^2 - 4ac$, and then determine whether one real-number solution, two different real-number solutions, or two different imaginary-number solutions exist.

$4x^2 = 8x + 5$

Find the zeros of the function algebraically. Give exact answers.

$f(x) = x^2 + 6x - 3$

Use a graphing calculator to find the zeros of the function. Round to three decimal places.

$f(x) = 9x^2 - 8x - 7$

Quadratic Types are those that canbe rewritten to be a quadratic after doing an appropriate u-substitution. Typically, we let u be equal to the middle variable part.

let u = middle variable part

Solve.

91. $x^4 - 3x^2 + 2 = 0$

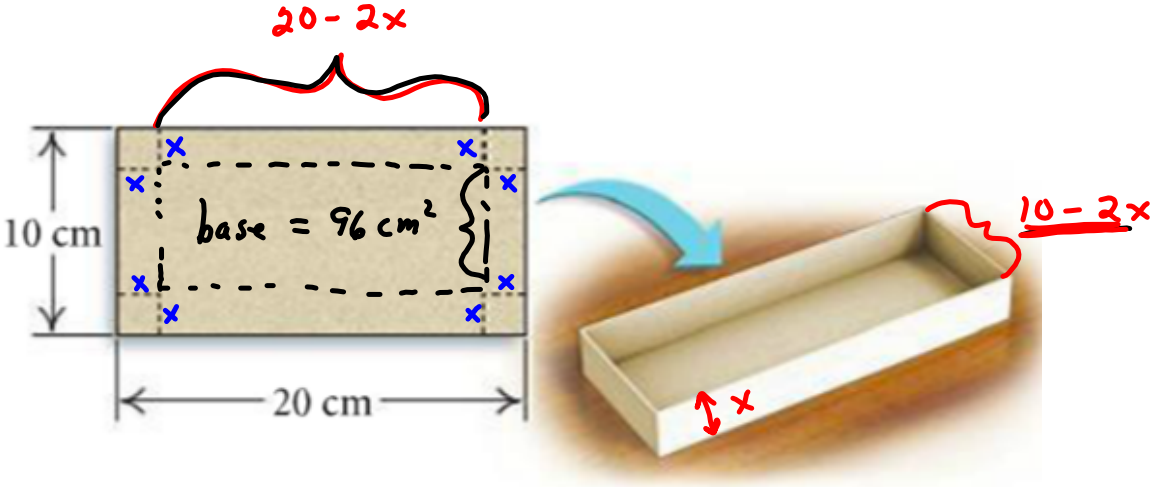
97. $x - 3\sqrt{x} - 4 = 0$
(*Hint*: Let $u = \sqrt{x}$.)

99. $m^{2/3} - 2m^{1/3} - 8 = 0$
(*Hint*: Let $u = m^{1/3}$.)

102. $x^{1/2} - 4x^{1/4} = -3$

103. $(2x - 3)^2 - 5(2x - 3) + 6 = 0$
(*Hint*: Let $u = 2x - 3$.)

117. *Box Construction.* An open box is made from a 10-cm by 20-cm piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



120. *Picture Frame Dimensions.* The frame on a picture is 8 in. by 10 in. outside and is of uniform width. What is the width of the frame if 48 in^2 of the picture shows?