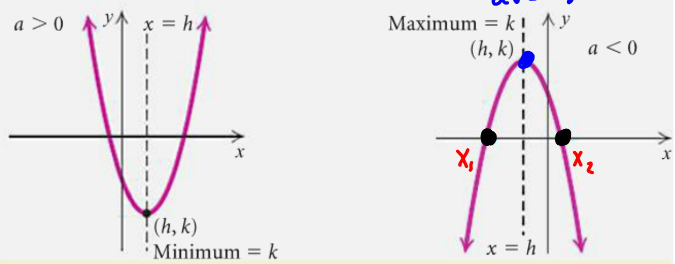
3.3 Analyzing Graphs of Quadratic Functions

Graphing Quadratic Functions of the Type $f(x) = ax^2 + bx + c, a \neq 0$

point (h, k) at which the graph turns is called the **vertex**. The maximum or minimum value of $f(x)$ occurs at the vertex. Each graph has a line $x = h$ that is called the **axis of symmetry**.

Graphing Quadratic Functions

Vertex is at the average of x_1 and x_2



$$X_1 = \frac{-b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$$

$$X_2 = \frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$$

$$\frac{X_1 + X_2}{2} = \frac{\frac{-2b}{2a}}{2} = \frac{-b}{2a}$$

THE VERTEX OF A PARABOLA

The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is

$$h \leftarrow \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

We calculate the x-coordinate. We substitute to find the y-coordinate.

In Exercises 3–16, **(a)** *find the vertex;* **(b)** *find the axis of symmetry;* **(c)** *determine whether there is a maximum or minimum value and find that value; and* **(d)** *graph the function.*

8. $f(x) = x^2 + 2x + 6$; $a = 1$, $b = 2$, $c = 6$ ← y-int

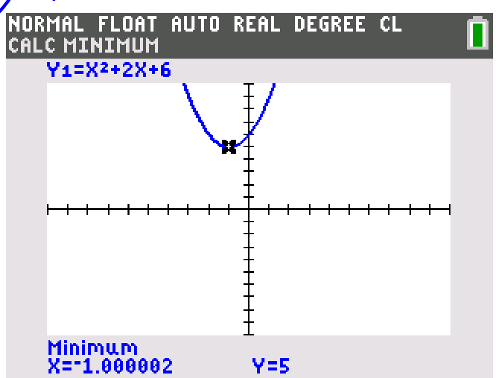
a) $h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$k = f(h) = f(-1) = (-1)^2 + 2(-1) + 6 = 1 - 2 + 6 = 5$

$V(-1, 5)$

b) $x = h$; $X = -1$

c) minimum is 5 at $X = -1$



10. $g(x) = \frac{x^2}{3} - 2x + 1$; $a = \frac{1}{3}$, $b = -2$, $c = 1$

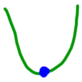
a) $h = \frac{-b}{2a} = \frac{-(-2)}{2(1/3)} = \frac{2}{2/3} = \frac{2}{1} \cdot \frac{3}{2} = 3$

$k = g(h) = g(3) = \frac{3^2}{3} - 2(3) + 1 = 3 - 6 + 1 = -2$

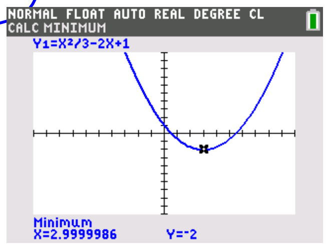
$V(3, -2)$

b) $x = h$; $x = 3$

c) $a > 0$



minimum of -2 at $x = 3$

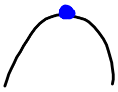


14. $f(x) = -x^2 - 8x + 5$; $a = -1$, $b = -8$, $c = 5$

a) $h = \frac{-b}{2a} = \frac{-(-8)}{2(-1)} = \frac{8}{-2} = -4$

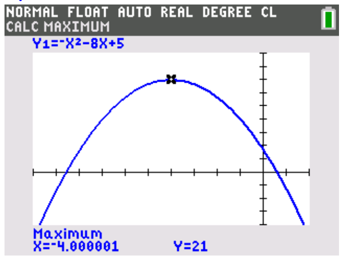
$k = f(-4) = -(-4)^2 - 8(-4) + 5 = -16 + 32 + 5 = 21$

$V(-4, 21)$



b) $x = -4$

c) $a < 0$ maximum of 21 at $x = -4$



In Exercises 31–40:

a) *Find the vertex.*

b) *Determine whether there is a maximum or minimum value and find that value.*

c) *Find the range.*

d) *Find the intervals on which the function is increasing and the intervals on which the function is decreasing.*

35. $f(x) = -\frac{1}{2}x^2 + 5x - 8$

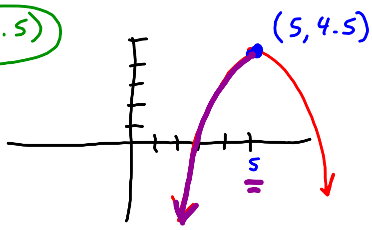
$a = -\frac{1}{2}$, $b = 5$, -8

a) $h = \frac{-b}{2a} = \frac{-5}{2(-\frac{1}{2})} = \frac{-5}{-1} = 5$

$k = -\frac{1}{2}(5)^2 + 5(5) - 8 = \frac{-25}{2} + \frac{50}{2} - \frac{16}{2} = \frac{9}{2} = 4.5$

$V(5, 4.5)$

b)



maximum of 4.5 at $x = 5$

c) $y \in (-\infty, 4.5]$

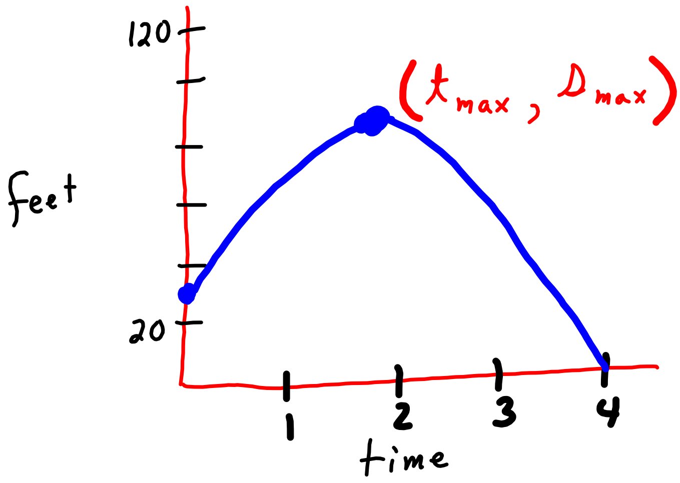
d) inc: $x \in (-\infty, 5)$
dec: $x \in (5, \infty)$

42. *Height of a Projectile.* A stone is thrown directly upward from a height of 30 ft with an initial velocity of 60 ft/sec. The height of the stone, in feet, t seconds after it has been thrown is given by the function $s(t) = -16t^2 + 60t + 30$. Determine the time at which the stone reaches its maximum height and find the maximum height.

$g = -32\ ft/sec^2$

$\approx -20\ mph/sec$

$s(t) = \frac{1}{2} g t^2 + v_0 t + s_0$



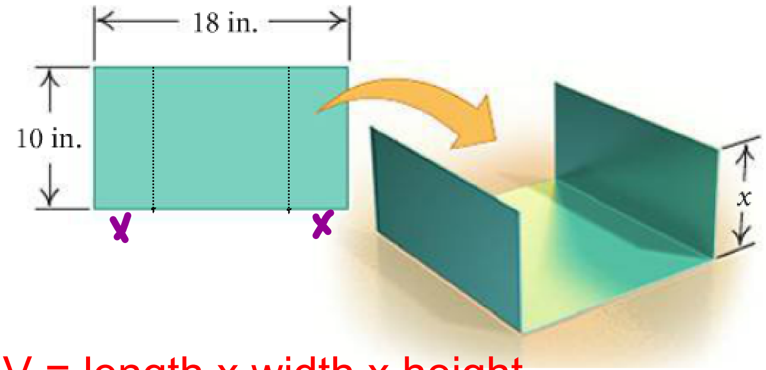
$t_{max} = \frac{-b}{2a} = \frac{-60}{2(-16)} = \frac{60}{32} = 1.875\ sec$

$s_{max} = s(1.875)$

$= -16(1.875)^2 + 60(1.875) + 30$

$= 86.25\ feet$

45. *Maximizing Volume.* Mendoza Manufacturing plans to produce a one-compartment vertical file by bending the long side of a 10-in. by 18-in. sheet of plastic along two lines to form a ⊔-shape. How tall should the file be in order to maximize the volume that it can hold?



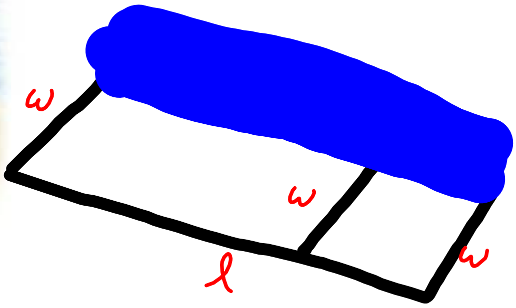
V = length x width x height

$V(x) = (18 - 2x)(10)(x)$

$V(x) = 20x(9 - x) = -20x^2 + 180x$; $a=-20$, $b=180$, $c=0$

$x_{max} = -b/(2a) = -180/-40 = 4.5$

53. *Maximizing Area.* A rancher needs to enclose two adjacent rectangular corrals, one for cattle and one for sheep. If a river forms one side of the corrals and 240 yd of fencing is available, what is the largest total area that can be enclosed?



$$3w + \ell = 240$$

$$\ell = 240 - 3w$$

$$A = w\,\ell$$

$$A(w) = w(240 - 3w) = -3w^2 + 240w$$

$$w_{max} = \frac{-b}{2a} = \frac{-240}{2(-3)} = \frac{240}{6} = 40 \text{ yards}$$

$$\ell_{max} = 240 - 3(40) = 120 \text{ yards}$$

$$A_{max} = (40\text{ yd})(120\text{ yd}) = 4800\text{ yd}^2$$