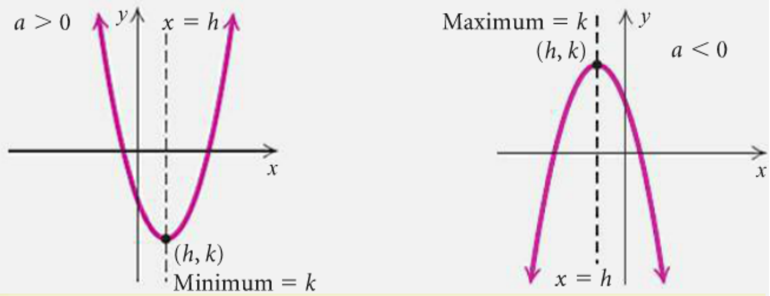
3.3 Analyzing Graphs of Quadratic Functions

Graphing Quadratic Functions of the Type $f(x) = ax^2 + bx + c, a \neq 0$

point (h, k) at which the graph turns is called the **vertex**. The maximum or minimum value of $f(x)$ occurs at the vertex. Each graph has a line $x = h$ that is called the **axis of symmetry**.

Graphing Quadratic Functions



THE VERTEX OF A PARABOLA

The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is

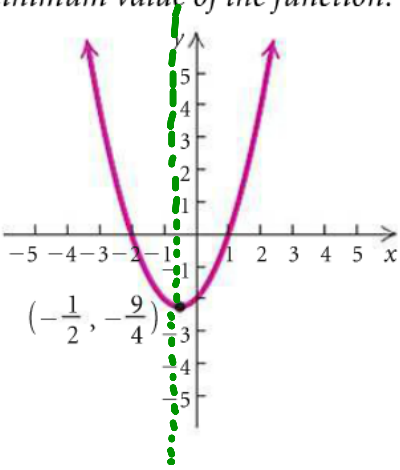
$$h \leftarrow \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

We calculate the x-coordinate. We substitute to find the y-coordinate.

$$f(x) = a(x-h)^2 + k$$
$$f(x) = a(x^2 - 2xh + h^2) + k$$
$$f(x) = ax^2 - 2ahx + (ah^2 + k)$$
$$f(x) = ax^2 + bx + c$$
$$b = -2ah \Rightarrow h = -\frac{b}{2a}$$
$$c = ah^2 + k$$

In Exercises 1 and 2, use the given graph to find **(a)** *the vertex;* **(b)** *the axis of symmetry; and* **(c)** *the maximum or minimum value of the function.*

1.

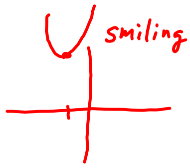


a) $V\left(-\frac{1}{2}, -\frac{9}{4}\right)$

b) $x = -\frac{1}{2}$

c) minimum value is $-\frac{9}{4}$

In Exercises 3–16, **(a)** *find the vertex;* **(b)** *find the axis of symmetry;* **(c)** *determine whether there is a maximum or minimum value and find that value; and* **(d)** *graph the function.*



8. $f(x) = x^2 + 2x + 6$; $a = 1,\ b = 2,\ c = 6$

(a) $h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

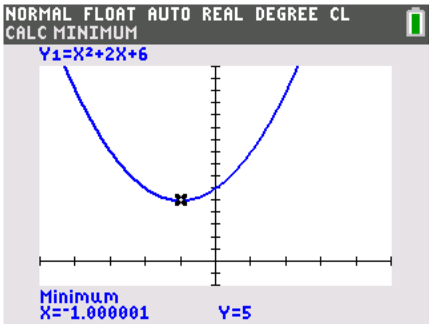
$k = f(h) = f(-1) = (-1)^2 + 2(-1) + 6 = 1 - 2 + 6 = 5$

$V(-1, 5)$

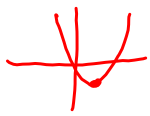
(b) $x = -1$

(c) 5 is the min

d)



10. $g(x) = \frac{x^2}{3} - 2x + 1$; $a = \frac{1}{3},\ b = -2,\ c = 1$



(a) $h = \frac{-b}{2a} = \frac{-(-2)}{2(\frac{1}{3})} = \frac{2}{\frac{2}{3}} = \frac{2}{1} \cdot \frac{3}{2} = 3$

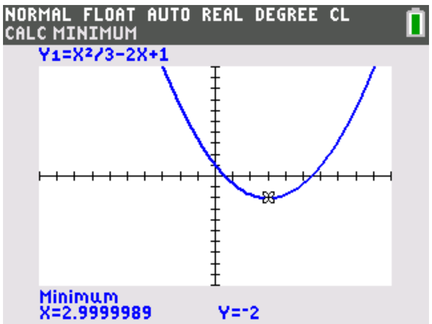
$k = g(h) = g(3) = \frac{(3)^2}{3} - 2(3) + 1 = 3 - 6 + 1 = -2$

$V(3, -2)$

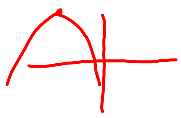
(b) $x = 3$

(c) -2 is the min

(d)



14. $f(x) = -x^2 - 8x + 5$; $a = -1,\ b = -8,\ c = 5$



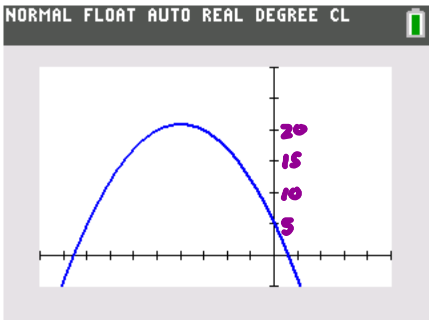
(a) $h = \frac{-b}{2a} = \frac{-(-8)}{2(-1)} = \frac{8}{-2} = -4$

$k = -(-4)^2 - 8(-4) + 5 = -16 + 32 + 5 = 21$

$V(-4, 21)$

(b) $x = -4$

(c) 21 is the max



In Exercises 31–40:

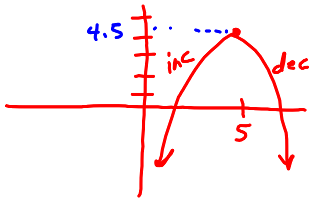
a) *Find the vertex.*

b) *Determine whether there is a maximum or minimum value and find that value.*

c) *Find the range.*

d) *Find the intervals on which the function is increasing and the intervals on which the function is decreasing.*

35. $f(x) = -\frac{1}{2}x^2 + 5x - 8$; $a = -\frac{1}{2}$, $b = 5$, $c = -8$



(a) $h = \frac{-b}{2a} = \frac{-5}{2(-\frac{1}{2})} = \frac{-5}{-1} = 5$

$k = -\frac{1}{2}(5)^2 + 5(5) - 8 = \frac{-25}{2} + \frac{50}{2} - \frac{16}{2} = \frac{9}{2} = 4.5$

$V(5, 4.5)$

(b) 4.5 is the max

(c) $y \in (-\infty, 4.5]$

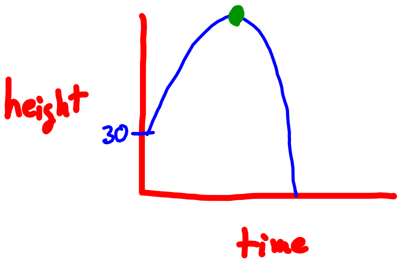
(d) inc: $x \in (-\infty, 5)$

dec: $x \in (5, \infty)$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

42. *Height of a Projectile.* A stone is thrown directly upward from a height of 30 ft with an initial velocity of 60 ft/sec. The height of the stone, in feet, t seconds after it has been thrown is given by the function $s(t) = -16t^2 + 60t + 30$. Determine the time at which the stone reaches its maximum height and find the maximum height.

$g = 9.8\,m/sec^2$
$= 32\,ft/sec^2$



$t_{max} = \frac{-b}{2a} = \frac{-60}{2(-16)} = \frac{60}{32} = \frac{15}{8}$

$= 1.875$ sec

max height $s(1.875) =$

$-16(1.875)^2 + 60(1.875) + 30 =$

86.25 ft

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NORMAL FLOAT AUTO REAL DEGREE CL
1.875→X
                              1.875
-16X²+60X+30
                              86.25
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