3.4 Radical Equations

Radical Equations

A **radical equation** is an equation in which variables appear in one or more radicands. For example,

$$\sqrt{2x-5} - \sqrt{x-3} = 1$$

is a radical equation. The following principle is used to solve such equations.

THE PRINCIPLE OF POWERS

For any positive integer n:

If $a = b$ is true, then $a^n = b^n$ is true.

Solve: $\sqrt{3x+1} = 4$.

We use the principle of powers and square both sides:

$$\sqrt{3x+1} = 4$$
$$\left(\sqrt{3x+1}\right)^2 = 4^2$$
$$3x+1 = 16$$
$$3x = 15$$
$$x = 5.$$

Check:

$\sqrt{3x+1}$	$= 4$
$\sqrt{3 \cdot 5 + 1}$	? 4
$\sqrt{15+1}$	
$\sqrt{16}$	
4	4 TRUE

The solution is 5.

38. $\left(\sqrt{2-7x}\right)^2 = 2^2$

$2 - 7x = 4$

$-2 \qquad -2$

$-7x = 2$

$x = -2/7$

$\checkmark$: $\sqrt{2 - 7(-2/7)} \stackrel{?}{=} 2$

$\sqrt{2+2} = 2$

$\sqrt{4} = 2$ true

54. $\sqrt[5]{2x-3}-1=1$

isolate $\quad +1 \quad +1$

$$\left(\sqrt[5]{2x-3}\right)^5 = 2^5$$

$$2x-3 = 32$$
$$+3 \quad +3$$

$$2x = 35$$

$$x = 17.5$$

not extraneous since 5 is odd

58. $\sqrt{x+3}-1=x$

isolate $\quad +1 \quad +1$

$$\left(\sqrt{x+3}\right)^2 = (x+1)^2$$

$$x+3 = x^2+2x+1$$
$$-x-3 \qquad -x-3$$

$$0 = x^2+x-2 \Rightarrow x^2+x-2=0$$

$$(x+2)(x-1)=0 \Rightarrow$$

$$\cancel{x=-2}, \; x=1$$

extraneous

$(a+b)^2 =$

$a^2+2ab+b^2$

Check:

-2	1
$\sqrt{-2+3}-1=-2$	$\sqrt{1+3}-1 \stackrel{?}{=} 1$
$\sqrt{1}-1=-2$	$\sqrt{4}-1=1$
$0=-2$	$2-1=1$
false	true

Find the real solutions of the equation.

$$\sqrt{5-4\sqrt{x}} = \sqrt{x}$$

$\Rightarrow 5 - 4\sqrt{x} = x$

$\Rightarrow -4\sqrt{x} = x - 5$

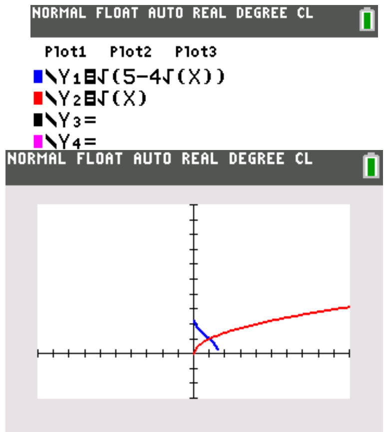
$\Rightarrow 16x = x^2 - 10x + 25$

$\Rightarrow x^2 - 26x + 25 = 0$

$\Rightarrow (x-25)(x-1) = 0$

$\Rightarrow x = 1, \cancel{25}$

Check 1	Check 25
$\sqrt{5-4} = \sqrt{1}$	$\sqrt{5-20} = \sqrt{25}$
$1 = 1$	
true	false



65. $\sqrt{x-3} + \sqrt{x+2} = 5$

$(a+b)^2 = a^2 + 2ab + b^2$

double

$$\left(\sqrt{x-3}\right)^2 = \left(5 - \sqrt{x+2}\right)^2$$

$$x - 3 = 25 - 10\sqrt{x+2} + x + 2$$

$$x - 3 = x - 10\sqrt{x+2} + 27$$

$-x \quad -27 \qquad -x \qquad -27$

$$-30 = -10\sqrt{x+2}$$

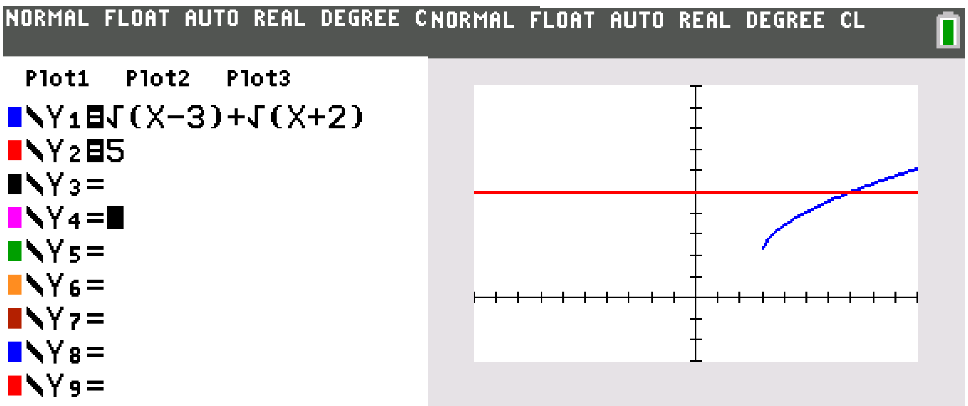
$$\sqrt{x+2} = 3 \Rightarrow x + 2 = 9$$

$$\Rightarrow x = 7$$

Check 7

$\sqrt{7-3} + \sqrt{7+2} = 5$

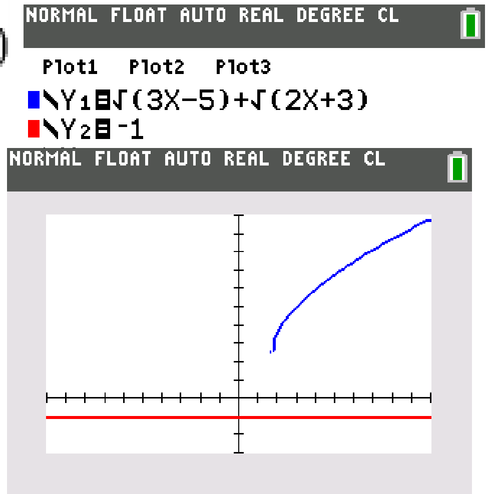
$2 + 3 = 5$ true



67. $\sqrt{3x-5} + \sqrt{2x+3} + 1 = 0$

$\sqrt{3x-5} + \sqrt{2x+3} = -1$

No Solution



$\sqrt[n]{x} = x^{\frac{1}{n}}$

78. $t^{1/5} = 2$

$\left(\sqrt[5]{t}\right)^5 = 2^5$

$t = 32$

80. $m^{1/2} = -7$

$\sqrt{m} = -7 \Rightarrow$ No Solution

$\cancel{m = 49}$

$W = \sqrt{\frac{1}{LC}}$, for C
(An electricity formula)

$$LCW^2 = \frac{1}{LC} LC$$

$$\frac{C \cdot L \cdot W^2}{LW^2} = \frac{1}{LW^2}$$

$$C = \frac{1}{LW^2}$$

isolate

$I = \sqrt{\frac{A}{P}} - 1$, for P
(A compound-interest formula)

$$(I+1)^2 = \left(\sqrt{\frac{A}{P}}\right)^2$$

$$P(I+1)^2 = \frac{A}{P} \cdot P$$

$$P \cdot (I+1)^2 = A$$

$$P = \frac{A}{(I+1)^2} \text{ OR } = \frac{A}{I^2+2I+1}$$

$T = 2\pi\sqrt{\frac{1}{g}}$, for g
(A pendulum formula)

$$g \cdot T^2 = 4\pi^2 \cdot \frac{1}{g} \cdot g$$

$$\frac{g \cdot T^2}{T^2} = \frac{4\pi^2}{T^2}$$

$$g = \frac{4\pi^2}{T^2}$$