3.4 Radical Equations

Radical Equations

A **radical equation** is an equation in which variables appear in one or more radicands. For example,

$$\sqrt{2x-5} - \sqrt{x-3} = 1$$

is a radical equation. The following principle is used to solve such equations.

THE PRINCIPLE OF POWERS

For any positive integer n:

If $a = b$ is true, then $a^n = b^n$ is true.

Solve: $\sqrt{3x+1} = 4$.

We use the principle of powers and square both sides:

$$\sqrt{3x+1} = 4$$
$$(\sqrt{3x+1})^2 = 4^2$$
$$3x+1 = 16$$
$$3x = 15$$
$$x = 5.$$

Check:

$$\sqrt{3x+1} = 4$$
$$\sqrt{3\cdot 5+1} \stackrel{?}{=} 4$$
$$\sqrt{15+1}$$
$$\sqrt{16}$$
$$4 \quad | \quad 4 \quad \text{TRUE}$$

The solution is 5.

38. $\left(\sqrt{2-7x}\right)^2 = (2)^2$

$2 - 7x = 4$

$-2 \qquad -2$

$-7x = 2$

$x = -2/7$

$\sqrt{}$: $\sqrt{2-7(-2/7)} \stackrel{?}{=} 2$

$\sqrt{2+2} = 2$

$\sqrt{4} = 2$ true

54. $\sqrt[5]{2x-3}-1=1$

isolate $+1 \quad +1$

$\left(\sqrt[5]{2x-3}\right)^5 = 2^5$

$2x-3=32$

$+3 \quad +3$

$2x=35$

$x=17.5$ not extraneous since 5 is odd

$(a+b)^2 = a^2+2ab+b^2$

58. $\sqrt{x+3}-1=x$

isolate $+1 \quad +1$

$\left(\sqrt{x+3}\right)^2 = (x+1)^2$

$x+3 = x^2+2x+1$

$-x-3 \quad -x-3$

$0 = x^2+x-2 \iff x^2+x-2=0$

$(x+2)(x-1)=0$

$x=\cancel{-2}$ extraneous, $x=1$

Check -2	Check 1
$\sqrt{-2+3}-1 \stackrel{?}{=} -2$	$\sqrt{1+3}-1 \stackrel{?}{=} 1$
$\sqrt{1}-1=-2$	$\sqrt{4}-1=1$
$0=-2$	$2-1=1$
false	true