3.5 Solving Equations and Inequalities with Absolute Value

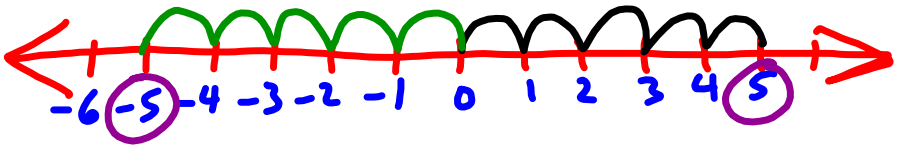
- Solve equations with absolute value.
- Solve inequalities with absolute value.

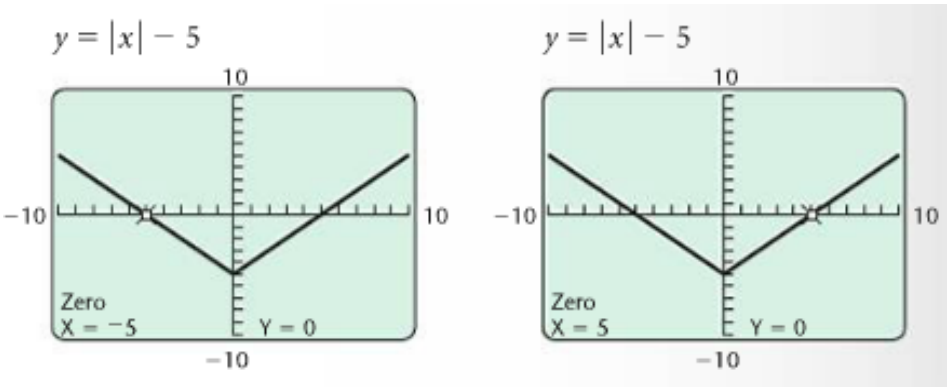
For $a > 0$ and an algebraic expression X:

$|X| = a$ is equivalent to $X = -a$ or $X = a$.

Say $a = 5$

$|x| = 5$





20. $\left|\frac{1}{3}x - 4\right| = 13$

$3\left(\frac{1}{3}x - 4\right) = 3(-13)$

$x - 12 = -39$

$x = -27$

$3\left(\frac{1}{3}x - 4\right) = 3(13)$

$x - 12 = 39$

$x = 51$

For $a > 0$ and an algebraic expression X:

$|X| = a$ is equivalent to $X = -a$ or $X = a$.

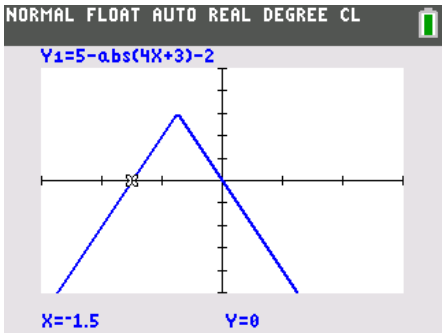
32. $5 - |4x + 3| = 2$

$-5 \quad$ isolate $\quad -5$

$-|4x+3| = -3$

$-1 \quad -1$

$|4x+3| = 3$



$4x+3 = -3$

$4x = -6$

$x = -3/2$

$4x+3 = 3$

$4x = 0$

$x = 0$

4) $|3x-7| = -4$

No Solution

For $a > 0$ and an algebraic expression X:

① $|X| < a$ is equivalent to $-a < X < a$.

② $|X| > a$ is equivalent to $X < -a$ or $X > a$.

Similar statements hold for ③ $|X| \leq a$ and ④ $|X| \geq a$.

Blind Date

< close the doors

> open the doors

① $|x| < a$

② $|x| > a$

③ $|x| \leq a$

④ $|x| \geq a$

① $(-a, a)$

② $(-\infty, -a) \cup (a, \infty)$

③ $[-a, a]$

④ $(-\infty, -a] \cup [a, \infty)$

For $a > 0$ and an algebraic expression X:

1. $|X| < a$ is equivalent to $-a < X < a$.
2. $|X| > a$ is equivalent to $X < -a$ or $X > a$.

Similar statements hold for 3. $|X| \leq a$ and 4. $|X| \geq a$.

Solve and graph the solution set: $|3x + 2| < 5$.

$-5 < 3x + 2 < 5$

$-2 \quad -2 \quad -2$

$\frac{-7}{3} < \frac{3x}{3} < \frac{3}{3}$

$\frac{-7}{3} < x < 1$ or $x \in \left(\frac{-7}{3}, 1\right)$

For $a > 0$ and an algebraic expression X:

1. $|X| < a$ is equivalent to $-a < X < a$.
2. $|X| > a$ is equivalent to $X < -a$ or $X > a$.

Similar statements hold for 3. $|X| \leq a$ and 4. $|X| \geq a$.

Solve and graph the solution set: $|5 - 2x| \geq 1$.

$5 - 2x \leq -1$ or $5 - 2x \geq 1$

$-2x \leq -6$ (flip) $-2x \geq -4$

$x \geq 3$ $x \leq 2$

$x \in (-\infty, 2] \cup [3, \infty)$