4.1 Polynomial Functions and Modeling

POLYNOMIAL FUNCTION

A **polynomial function** P is given by

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0,$$

where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

POLYNOMIAL FUNCTION	EXAMPLE	DEGREE	LEADING TERM	LEADING COEFFICIENT
Constant	$f(x) = 3 \quad (f(x) = 3 = 3x^0)$	0	3	3
Linear	$f(x) = \frac{2}{3}x + 5 \quad \left(f(x) = \frac{2}{3}x + 5 = \frac{2}{3}x^1 + 5\right)$	1	$\frac{2}{3}x$	$\frac{2}{3}$
Quadratic	$f(x) = 4x^2 - x + 3$	2	$4x^2$	4
Cubic	$f(x) = x^3 + 2x^2 + x - 5$	3	x^3	1
Quartic	$f(x) = -x^4 - 1.1x^3 + 0.3x^2 - 2.8x - 1.7$	4	$-x^4$	-1

Determine the a) leading term, b) the leading coefficient, and the c) degree of the polynomial. Then d) classify the polynomial function as constant, linear, quadratic, cubic, or quartic.

2. $f(x) = 15x^2 - 10 + \underline{0.11x^4} - 7x^3$

a) $0.11x^4$

b) 0.11

c) 4

d) quartic

7. $h(x) = -5x^2 + 7x^3 + x^4$

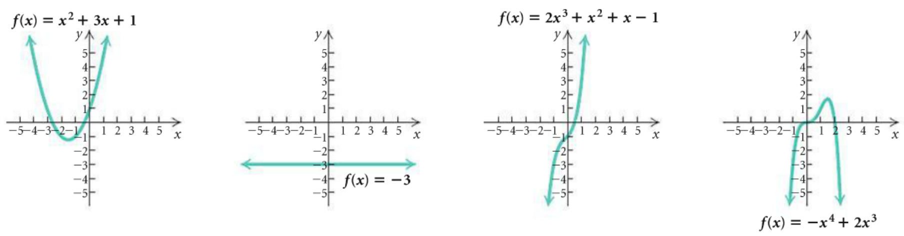
a) x^4

b) 1

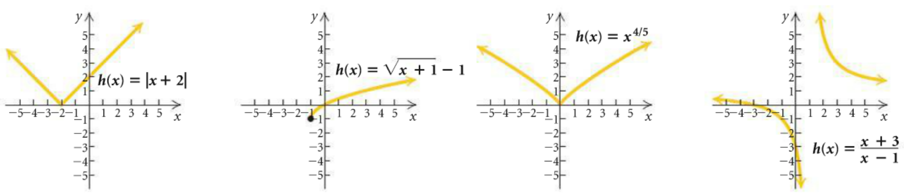
c) 4

d) quartic

Polynomial Functions

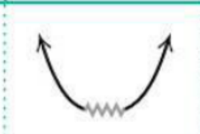
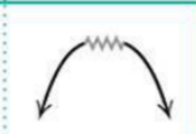
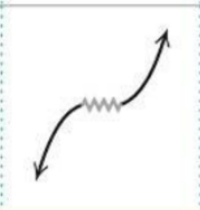


Nonpolynomial Functions



THE LEADING-TERM TEST

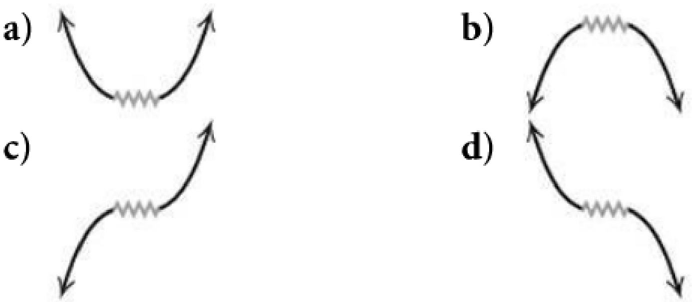
If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ or as $x \to -\infty$ can be described in one of the four following ways.

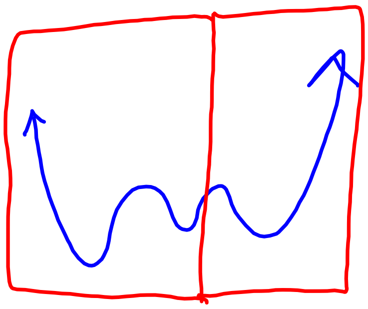
n	$a_n > 0$	$a_n < 0$
Even		
Odd		

The portion of the graph is not determined by this test.

The *domain* of a polynomial function is the set of all real numbers, $(-\infty, \infty)$.

In Exercises 11–18, select one of the four sketches (a)–(d), which follow, to describe the end behavior of the graph of the function.





12. $f(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 6x^2 + x - 5$ (a)

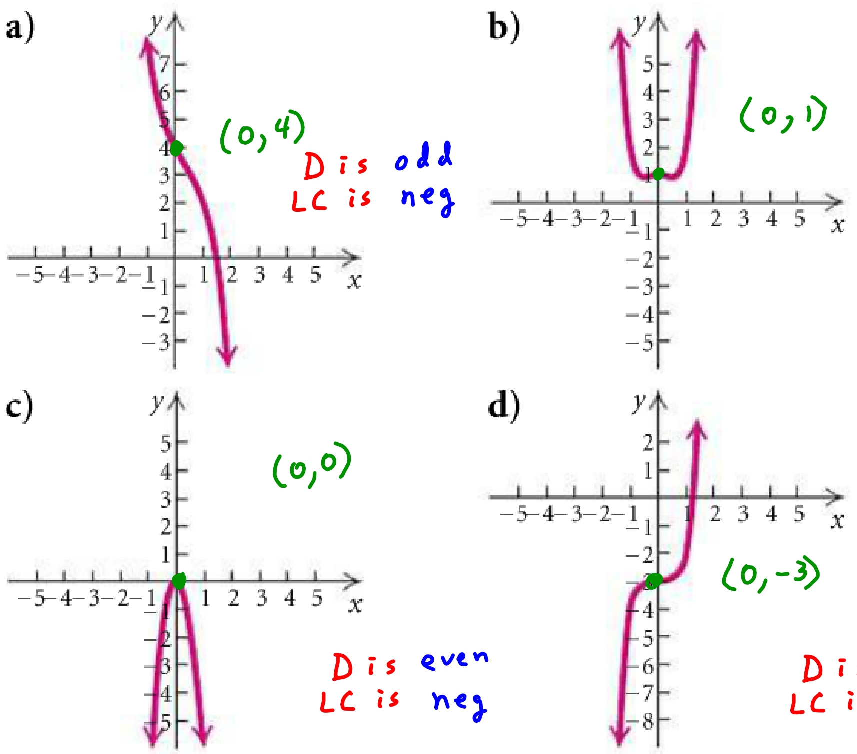
14. $f(x) = \frac{2}{5}x^5 - 2x^4 + x^3 - \frac{1}{2}x + 3$ (c)

15. $f(x) = -3.5x^4 + x^6 + 0.1x^7$ (c)

18. $f(x) = 2x + x^3 - x^5$ (d)

In Exercises 19–22, use the leading-term test to match the function with one of the graphs (a)–(d), which follow.

D = degree
LC = lead coefficient



D is even
LC is pos

19. $f(x) = -x^6 + 2x^5 - 7x^2$; $f(0) = 0$ c

20. $f(x) = 2x^4 - x^2 + 1$; $f(0) = 1$ b

21. $f(x) = x^5 + \frac{1}{10}x - 3$; $f(0) = -3$ d

22. $f(x) = -x^3 + x^2 - 2x + 4$; $f(0) = 4$ a

EVEN AND ODD MULTIPLICITY

If $(x - c)^k$, $k \geq 1$, is a factor of a polynomial function $P(x)$ and $(x - c)^{k+1}$ is not a factor and:

- k is odd, then the graph crosses the x-axis at $(c, 0)$;
- k is even, then the graph is tangent to the x-axis at $(c, 0)$. That is, it will bounce off the x-axis.

$x - 1 = 0 \Rightarrow x = 1$

$2x + 3 = 0 \Rightarrow x = -3/2$

Example: $f(x) = \frac{1}{3}(x-1)(2x+3)^2$

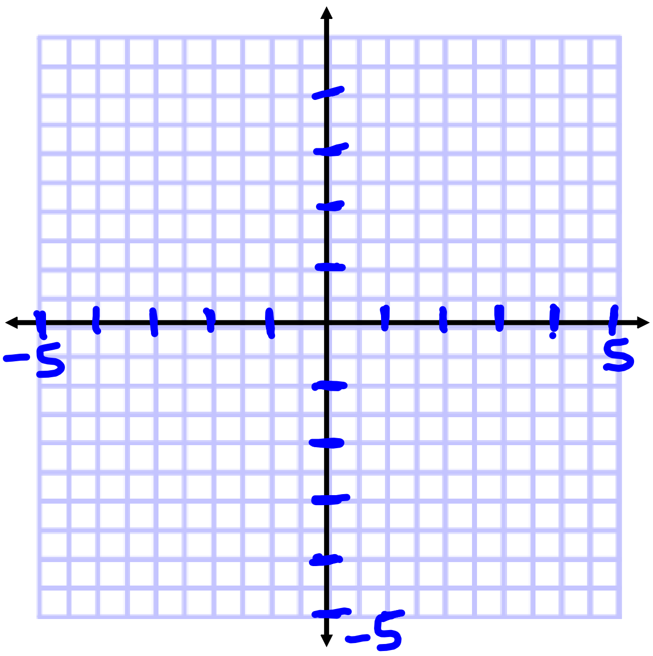
is a zero of multiplicity

is a zero of multiplicity

What is the degree of $f(x)$?

What is the leading coefficient?

What is its endpoint behavior?



$$f(x) = \frac{1}{3}(x-1)(4x^2 + 12x + 9)$$
$$= \frac{1}{3}(4x^3 + 12x^2 + 9x - 4x^2 - 12x - 9)$$
$$= \frac{1}{3}(4x^3 + 8x^2 - 3x - 9)$$
$$= \frac{4}{3}x^3 + \frac{8}{3}x^2 - x - 3$$