4.3 Polynomial Division; The Remainder Theorem and the Factor Theorem

Divide to determine whether $\underline{x + 1}$ and $\underline{x - 3}$ are factors of P(x)= $x^3 + 2x^2 - 5x - 6$.

Solution We divide $x^3 + 2x^2 - 5x - 6$ by $x + 1$.

$$
\begin{array}{rl}
 & \overbrace{x^2 + x - 6}^{\text{Quotient}} \\
\underbrace{x + 1}_{\text{Divisor}} & \overline{)\,x^3 + 2x^2 - 5x - 6} \leftarrow \text{Dividend} \\
 & \underline{x^3 + x^2} \\
 & \quad x^2 - 5x \\
 & \quad \underline{x^2 + x} \\
 & \qquad -6x - 6 \\
 & \qquad \underline{-6x - 6} \\
 & \qquad\qquad 0 \leftarrow \text{Remainder}
\end{array}
$$

this is the zero of the divisor, x+1.

	x^3	x^2	x^1	x^0
-1	1	2	-5	-6
	0	-1	-1	6
	1	1	-6	0

coefficients of quotient

remainder

If r=0, then divisor is a factor.

$Q(x) = 1x^2 + 1x - 6$

$= x^2 + x - 6$

Instant Replay

-1	1	2	-5	-6

Is x-3 a factor?

Divide to determine whether $x + 1$ and $x - 3$ are factors of P(x)= $x^3 + 2x^2 - 5x - 6$.

$$x+1 \overline{\big)\, x^3 + 2x^2 - 5x - 6}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & & & \\ \hline & & & & \end{array}$$

Is x-3 a factor?

THE REMAINDER THEOREM

If a number c is substituted for x in the polynomial $f(x)$, then the result $f(c)$ is the remainder that would be obtained by dividing $f(x)$ by $x - c$. That is, if $f(x) = (x - c) \cdot Q(x) + R$, then $f(c) = R$.

$$\underbrace{P(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}} \cdot \underbrace{Q(x)}_{\text{Quotient}} + \underbrace{R(x)}_{\text{Remainder}}$$

Easy Example:

$$\begin{array}{r|l} & \ 7 \text{ quotient} \\ \text{divisor } 5 & \ 37 \text{ dividend} \\ & \ \underline{35} \\ & \quad 2 \text{ remainder} \end{array} \iff 37 = (5)(7) + 2$$

P(x)= $x^3 + 2x^2 - 5x - 6.$

$$\begin{array}{r|rrrr} 3 & 1 & 2 & -5 & -6 \\ & \downarrow & 3 & 15 & 30 \\ \hline & 1 & 5 & 10 & \boxed{24} \end{array}$$

$$\begin{array}{r|l} & \quad x^2 + 5x + 10 \\ x - 3 & x^3 + 2x^2 - 5x - 6 \\ \hline & \qquad r = 24 \end{array}$$

$$P(x) = x^3 + 2x^2 - 5x - 6$$
$$= (x-3)(x^2 + 5x + 10) + 24$$

$P(3) = 24$; the remainder when the divisor was $x - 3$

If $P(x) = -2x^6 + 9x^4 - x^3 + x^2 - 16$, find $P(-2)$

Of Course, $P(-2) =$

$-2(-2)^6 + 9(-2)^4 - (-2)^3 + (-2)^2 - 16 =$

$-2(64) + 9(16) - (-8) + 4 - 16 =$

$-128 + 144 + 8 + 4 - 16 =$

$(144 + 8 + 4) - (128 + 16) =$

$156 - 144 = \boxed{12}$

Note: $P(-2) =$ remainder when the divisor is $x+2$.

-2	-2	0	9	-1	1	0	-16
	0	4	-8	-2	6	-14	28
	-2	4	1	-3	7	-14	12

Instant Replay

-2	-2	0	9	-1	1	0	-16

In each of the following, a polynomial $P(x)$ and a divisor $d(x)$ are given. ~~Use long division to~~ find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $d(x)$. Express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$.

5. $P(x) = x^3 - 8,$
$d(x) = x + 2$

8. $P(x) = x^3 - 9x^2 + 15x + 25,$
$d(x) = x - 5$

Use synthetic division to find the quotient and the remainder.

22. $(3x^4 - 2x^2 + 2) \div \left(x - \frac{1}{4}\right)$

Use synthetic division to find the function values. Then check your work using a graphing calculator.

29. $f(x) = x^3 - 5x - 2$; find $f(\cancel{2})$, $f(\cancel{-2})$, $f(\cancel{3})$, and $f(1 - \sqrt{2})$.

$$\begin{array}{c|cccc} 1-\sqrt{2} & 1 & 0 & -5 & -2 \\ & & & & \\ \hline & & & & \end{array}$$

$$(1-\sqrt{2})(1-\sqrt{2}) = 1 - \sqrt{2} - \sqrt{2} + 2 = 3 - 2\sqrt{2}$$

$$(1-\sqrt{2})(-2-2\sqrt{2}) = -2 - 2\sqrt{2} + 2\sqrt{2} + 4 = 2$$

$$Q(x) = 1x^2 + (1-\sqrt{2})x - 2 - 2\sqrt{2}$$

Claim: $1+\sqrt{2}$ is a zero for $Q(x)$

$$\begin{array}{c|ccc} 1+\sqrt{2} & 1 & 1-\sqrt{2} & -2-2\sqrt{2} \\ & & & \\ \hline & & & \end{array}$$

Are 2, 7, and 13 factors of 238?

Reuse Quotients

$2\overline{)238}$: 119; 2, 3, 2, 18, 18, 0 — Yes

$7\overline{)238}$: 34; 21, 28, 28, 0 — Yes

$13\overline{)238}$: 18; 13, 108, 104, 4 — No

$2\overline{)238}$: 119; 2, 3, 2, 18, 18, 0 — Yes

$7\overline{)119}$: 17; 7, 49, 49, 0 — Yes

$13\overline{)17}$: 1; 13, 4 — No

Using synthetic division, determine whether the numbers are zeros of the polynomial function.

31. $-3, 2;\ f(x) = 3x^3 + 5x^2 - 6x + 18$

38. $i, -i, -2;\ f(x) = x^3 + 2x^2 + x + 2$

factoid: if P(x) is a polynomial with real coefficient and a + bi is a zero then its conjugate a - bi is also a zero.