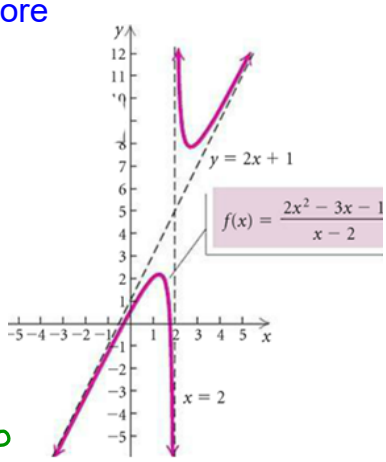
Oblique Asymptotes

Sometimes a line that is neither horizontal nor vertical is an asymptote. Such a line is called an **oblique asymptote**, or a **slant asymptote**.

The degree of the numerator must be exactly one more than the degree of the denominator for an oblique asymptote to exist.



a) -oblique

Find all the asymptotes of $f(x) = \dfrac{2x^2 - 3x - 1}{x - 2}$.

$$x-2 \overline{)\,2x^2 - 3x - 1}$$

$2x + 1$

$2x^2 - 4x$

$x - 1$

$x - 2$

1

$f(x) = 2x + 1 + \dfrac{1}{x-2}$

$\dfrac{1}{x-2}$ never 0

as $x \to \pm\infty$, $f(x) \to 2x+1$

$$2 \overline{)\,2 \quad -3 \quad -1}$$

$4 \quad 2$

$2 \quad 1 \quad 1$

OA: $y = 2x + 1$

b) $f(x) = \dfrac{7x^2 - 9x + 8}{2x}$

OA: $y = \frac{7}{2}x - \frac{9}{2}$

$= \dfrac{7x^2}{2x} - \dfrac{9x}{2x} + \dfrac{8}{2x}$

$= \frac{7}{2}x - \frac{9}{2} + \frac{4}{x}$

c) $f(x) = \dfrac{8x^4 - 7x + 5}{2x^3 + x^2}$

$$2x^3 + x^2 + 0x + 0 \overline{)\,8x^4 + 0x^3 + 0x^2 - 7x + 5}$$

$4x - 2 + \dfrac{-2x^2 - 7x + 5}{2x^3 + x^2}$

$8x^4 + 4x^3 + 0x^2 + 0x$

$-4x^3 + 0x^2 - 7x + 5$

$-4x^3 - 2x^2 + 0x + 0$

$-2x^2 - 7x + 5$

Find the oblique asymptote

28. $f(x) = \dfrac{x^2 - 6x}{x - 5}$

5	1	-6	0
		5	-5
	1	-1	-5

OA: $y = x - 1$

$$f(x) = \frac{(x+2)^2}{2x} = \frac{x^2 + 4x + 4}{2x}$$ monomial

$$= \frac{x^2}{2x} + \frac{4x}{2x} + \frac{4}{2x}$$

$$= \frac{1}{2}x + 2 + \frac{4}{2x}$$

OA: $y = \frac{1}{2}x + 2$

Sketch

27. $g(x) = \dfrac{x^2 + 4x - 1}{x + 3}$

NORMAL FLOAT AUTO REAL DEGREE CL

X=
-2+/-√5

PRESS ENTER
WHEN FINISHED

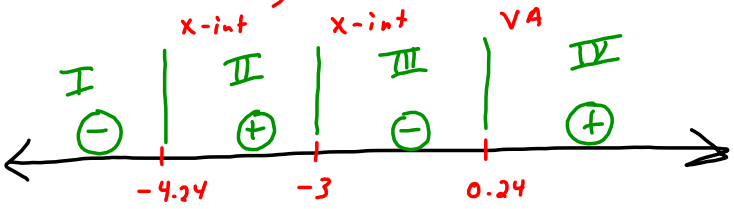
① x-int: $(-2-\sqrt{5}, 0) \approx (-4.24, 0)$
$(-2+\sqrt{5}, 0) \approx (0.24, 0)$

② y-int: $g(0) = -\frac{1}{3}$ $\quad (0, -\frac{1}{3})$

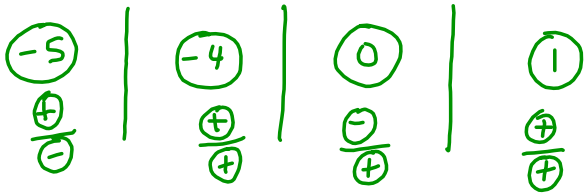
③ VA: $x = -3$
HA: does not exist
OA: $y = x + 1$ $\quad m = 1 = \frac{1}{1}$, $b = 1$

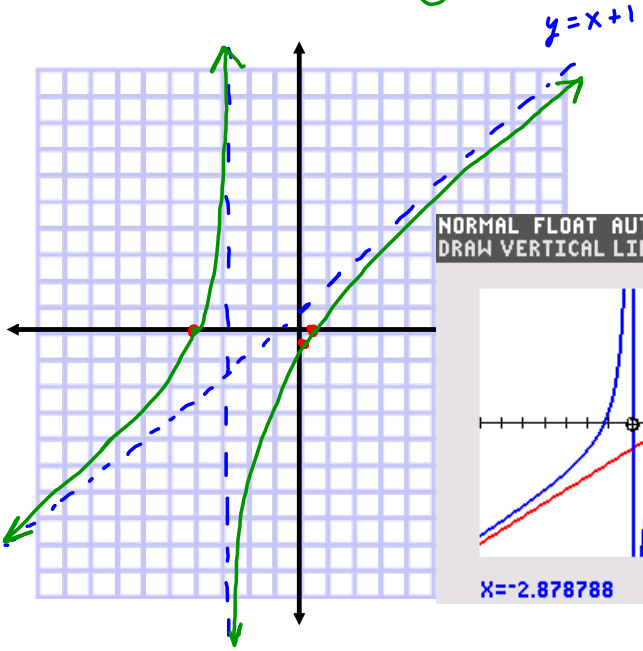
	1	4	−1
−3		−3	−3
	1	1	−4

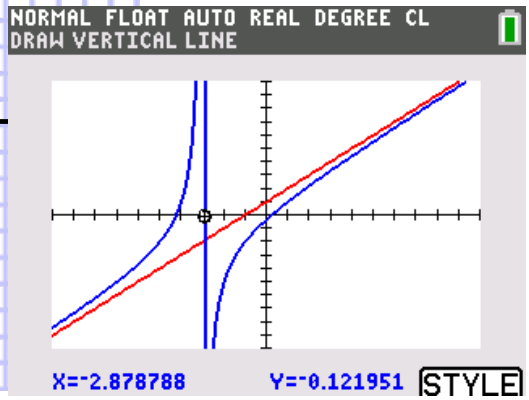
④ CV: -4.24, 0.24, -3



TV







Find a rational function that satisfies the given conditions. Answers may vary, but try to give the simplest answer possible.

81. Vertical asymptotes $x = -4$, $x = 5$; horizontal asymptote $y = \frac{3}{2}$; x-intercept $(-2, 0)$

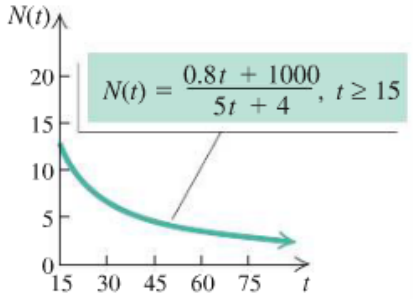
$$f(x) = \frac{3(x+2)^2}{2(x+4)(x-5)}$$

Applications

Medical Dosage. The function

$$N(t) = \frac{0.8t + 1000}{5t + 4}, \quad t \geq 15,$$

gives the body concentration $N(t)$, in parts per million, of a certain dosage of medication after time t, in hours.



a) Find the horizontal asymptote of the graph and complete the following:

$N(t) \longrightarrow$ ____ as $t \longrightarrow \infty$.

HA: $N(t) \to \frac{0.8}{5} = .16$

b) Explain the meaning of the answer to part (a) in terms of the application.

There will always be a trace of the medication in your body.