Rational Functions 4.5

Objectives:

Find the domain, intercepts, and asymptotes for a rational function, and graph the function.

Solve applications.

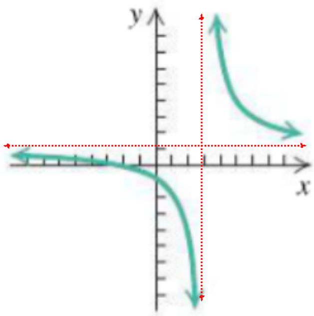
RATIONAL FUNCTION

A **rational function** is a function f that is a quotient of two polynomials. That is,

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomials and where $q(x)$ is not the zero polynomial. The domain of f consists of all inputs x for which $q(x) \neq 0$.

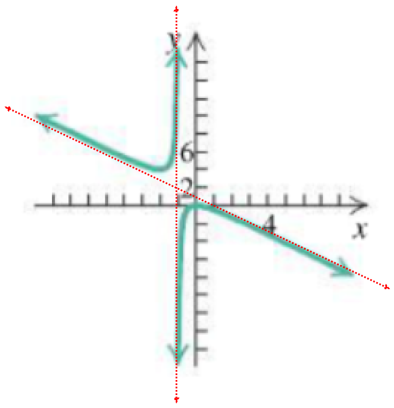
$$f(x) = \frac{2x+5}{2x-6}$$



Vertical Asymptote at $x = 3$

Horizontal Asymptote at $y = 1$

$$f(x) = \frac{-x^2}{x+1}$$

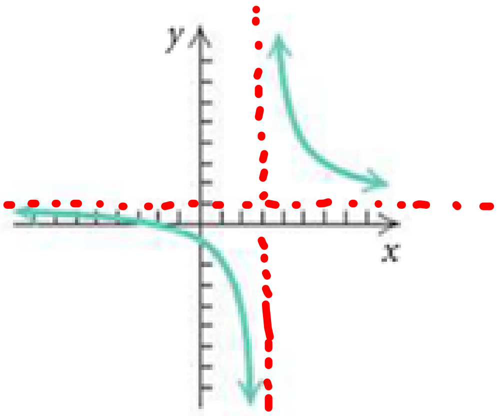


Vertical Asymptote at $x = -1$

Oblique Asymptote at $y = -x + 1$

An **asymptote** is something you come close to but never touch or never touch again after a particular point.

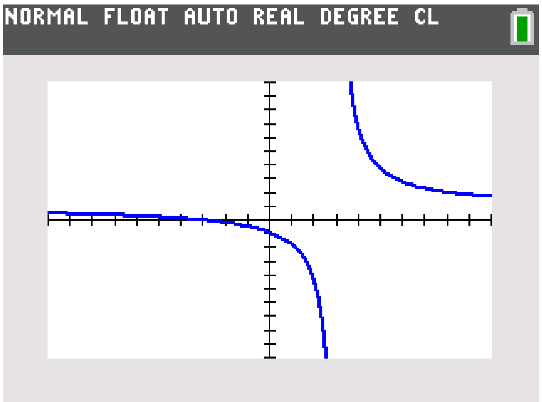
$$f(x) = \frac{2x+5}{2x-6}$$



NORMAL FLOAT AUTO REAL DEGREE CL

Plot1 Plot2 Plot3

\Y1=(2X+5)/(2X-6)



A function has a vertical asymptote at $x = c$ if $f(c)$ does not exist and

$$\lim_{x \to c} f(x) = \infty \quad \text{or}$$

$$\lim_{x \to c} f(x) = -\infty.$$

A function has a horizontal asymptote at $y = k$ if

$$\lim_{x \to \infty} f(x) = k \quad \text{or}$$

$$\lim_{x \to -\infty} f(x) = k$$

NORMAL FLOAT AUTO REAL DEGREE CL

X	Y1			
3	ERROR			
2.9	-54			
2.988	-457.3			
2.994	-915.7			
3.004	1376			
3.001	5501			
1000	1.0055			
6500	1.0008			
-2500	0.9978			
-9999	0.9995			

X=

DETERMINING VERTICAL ASYMPTOTES

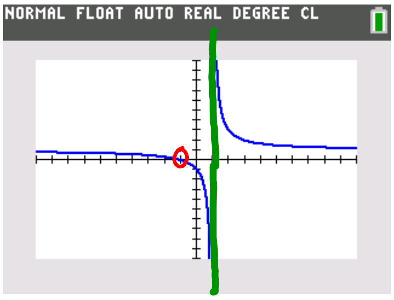
For a rational function $f(x) = p(x)/q(x)$, where $p(x)$ and $q(x)$ are polynomials with no common factors other than constants, if a is a zero of the denominator, then the line $x = a$ is a vertical asymptote for the graph of the function.

That is, if f(x) is reduced, they are the real Zeros of the denominator.

Determine the vertical asymptotes

If the value is not in the domain, it is either a vertical asymptote or a hole in the graph.

① $f(x) = \dfrac{x^2+2x+1}{x^2-1} = \dfrac{(x+1)(x+1)}{(x+1)(x-1)} = \dfrac{x+1}{x-1}$

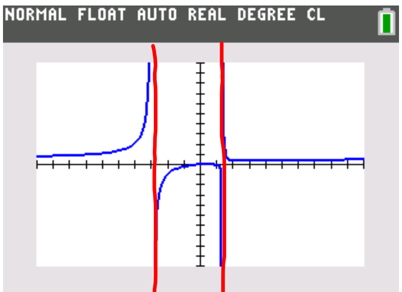


$x=1$

VA: $x=1$

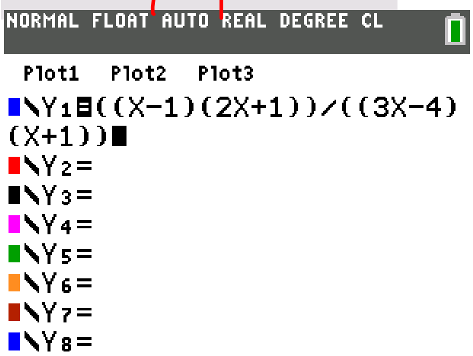
domain: $x \neq -1, 1$

② $f(x) = \dfrac{(x-1)(2x+1)}{(3x-4)(x+3)}$

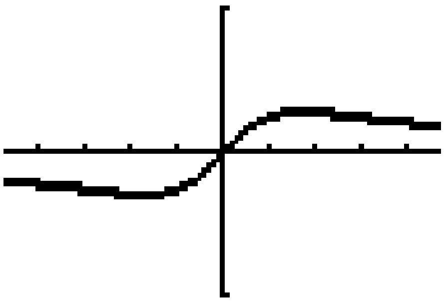


VA: $3x-4=0 \Rightarrow x = 4/3$

$x+3=0 \Rightarrow x=-3$



③ $f(x) = \dfrac{x}{x^2+3}$



VA: $x^2 + 3 = 0$

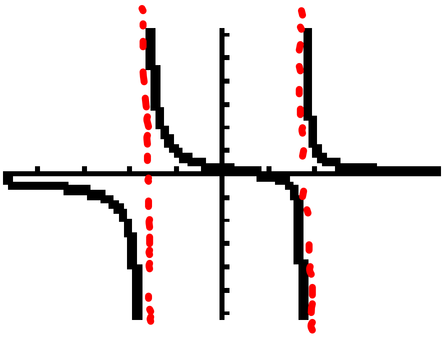
$x^2 = -3$

$x = \pm\sqrt{-3} = \pm\sqrt{3}\,i$

nonreal

VA does not exist

④ $f(x) = \dfrac{x-1}{x^2-3}$



VA: $x^2 - 3 = 0$

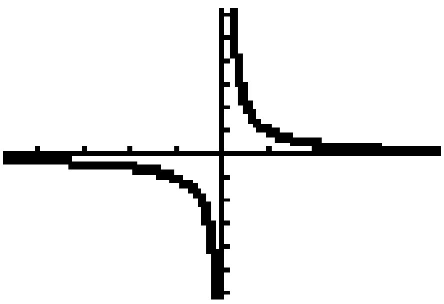
$x^2 = 3$

$x = \pm\sqrt{3}$

VA: $x = \sqrt{3} \approx 1.7$

$x = -\sqrt{3} \approx -1.7$

⑤ $f(x) = \dfrac{1}{x}$



VA: $x = 0$ or

y-axis