5.2 Exponential Functions and Graphs

- Graph exponential equations and exponential functions.
- Solve applied problems involving exponential functions and their graphs.

EXPONENTIAL FUNCTION

The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called the **exponential function, base *a*.**

EXAMPLE $y = f(x) = 2^x$

$f(0) = 2^0 = 1;$ $f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2};$

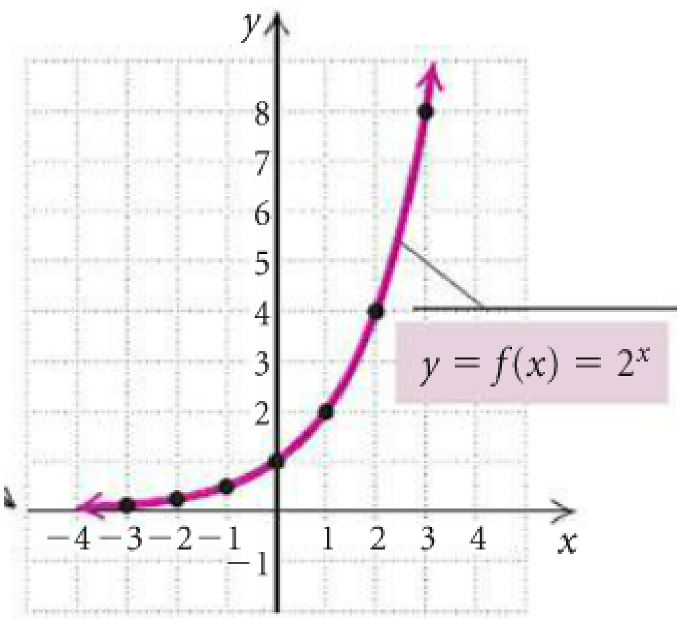
$f(1) = 2^1 = 2;$

$f(2) = 2^2 = 4;$ $f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4};$

$f(3) = 2^3 = 8;$

$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$

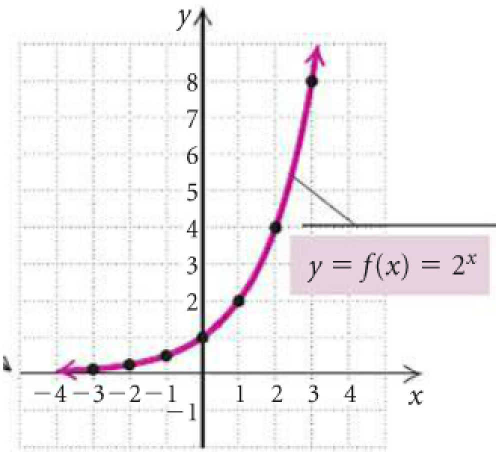
x	y $y = f(x) = 2^x$	(x, y)
0	1	$(0, 1)$
1	2	$(1, 2)$
2	4	$(2, 4)$
3	8	$(3, 8)$
-1	$\frac{1}{2}$	$(-1, \frac{1}{2})$
-2	$\frac{1}{4}$	$(-2, \frac{1}{4})$
-3	$\frac{1}{8}$	$(-3, \frac{1}{8})$

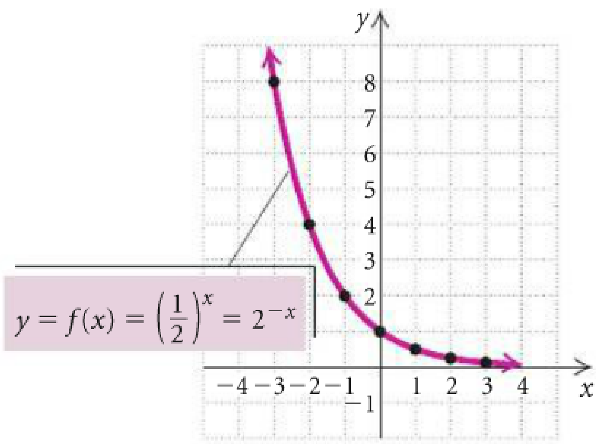


EXAMPLE $y = f(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$

x	y $y = f(x) = 2^x$	(x, y)
0	1	$(0, 1)$
-1	$\frac{1}{2}$	$\left(-1, \frac{1}{2}\right)$
-2	$\frac{1}{4}$	$\left(-2, \frac{1}{4}\right)$
-3	$\frac{1}{8}$	$\left(-3, \frac{1}{8}\right)$
1	2	$(1, 2)$
2	4	$(2, 4)$
3	8	$(3, 8)$

x	y $y = f(x) = \left(\frac{1}{2}\right)^x$	(x, y)
0	1	$(0, 1)$
-1	2	$(-1, 2)$
-2	4	$(-2, 4)$
-3	8	$(-3, 8)$
1	$\frac{1}{2}$	$\left(1, \frac{1}{2}\right)$
2	$\frac{1}{4}$	$\left(2, \frac{1}{4}\right)$
3	$\frac{1}{8}$	$\left(3, \frac{1}{8}\right)$





<u>Properties of exponential functions of the form $y=a^x$</u>

(1) y-intercept at (0,1)

(2) Domain is all real numbers, Range is all positive real numbers.

(3) The x-axis is a horizontal asymptote; never crosses x-axis.

(4) If a > 1, then it is an increasing function

(5) If 0 < a < 1, then it is a decreasing function

(6) It is a one-to-one function. It's inverse function is a logarithm.

Most important number in the universe

Most famous exponential function is

$f(x) = e^x$, where e is the euler number (pronounced "oiler")

$$e = \lim_{k \to \infty}\left(1 + \frac{1}{k}\right)^k \approx 2.718281828459$$

```
(1+1/100)^100
     2.704813829
```

```
(1+1/1000)^1000
     2.716923932
```

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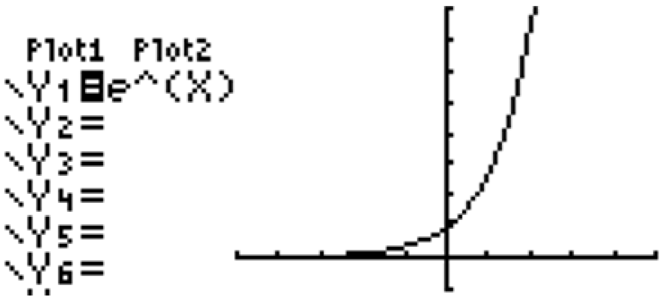
(1+1/1000000)^10
00000
     2.718280469
```



Find each of the following, to four decimal places, using a calculator.

1. e^4

3. $e^{-2.458}$

Simple versus Compounded Interest

$I = Prt$; $F = P + Prt = P(1+rt)$

\$100 investment at 10% ($P = 100$, $r = .1$)

Simple	Compounded Annually
Year 0	
Year 1	
Year 2	
Year 3	
⋮	
Year 50	

EXAMPLE *Compound Interest.* The amount of money A to which a principal P will grow after t years at interest rate r (in decimal form), compounded n times per year, is given by the function

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

If $35,000 is invested at 6.5% for 15 years, what will be its future value if it is compunded?

(a) Annually (n=1)
(b) Semiannually (n=2)
(c) Monthly (n=12)
(d) Weekly (n=52)
(e) Daily (n=365)
(f) Continouly (n approaches infinity)
(g) If compunded continuously, how long will it take for the investment to be $100,000?

a)
```
35000(1+.065)^(1
5)
            90014.43523
```

b)
```
35000(1+.065/2)^
(2*15)
            91362.89535
```

c)
```
35000(1+.065/12)
^(12*15)
            92547.02868
```

d)
```
35000(1+.065/52)
^(52*15)
            92734.37223
```

e)
```
35000(1+.065/365
)^(365*15)
            92782.79806
```

$e = \lim_{k \to \infty}\left(1+\frac{1}{k}\right)^k \approx 2.718281828459$

f) Continouly (n approaches infinity)

$$A(t) = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}$$

$$k = \frac{n}{r} \Rightarrow k \to \infty, \frac{1}{k} = \frac{r}{n}, n = kr$$

$$A(t) = \lim_{n\to\infty} P\left(1+\frac{1}{k}\right)^{krt} = \lim_{n\to\infty} P\left[\left(1+\frac{1}{k}\right)^{k}\right]^{rt} = Pe^{rt}$$

```
35000e^(.065*15)
        92790.85238
```

g)

```
Plot1 Plot2 Plot3
\Y1=35000e^(.065
X)
\Y2=100000
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=5
 Ymin=0
 Ymax=125000
 Yscl=25000
↓Xres=
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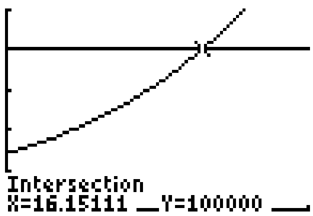

52. *Compound Interest.* Suppose that $750 is invested at 7% interest, compounded semiannually.

a) Find the function for the amount to which the investment grows after t years.

b) Graph the function.

c) Find the amount of money in the account at $t = 1, 6, 10, 15$, and 25 years.

d) When will the amount of money in the account reach $3000?

63. *Overweight Troops.* The number of U.S. troops diagnosed as overweight has more than doubled since 1998 (*Source*: U.S. Department of Defense). The exponential function

$$W(x) = 23{,}672.16(1.112)^x,$$

where x is the number of years since 1998, can be used to estimate the number of overweight active troops. Find the number of overweight troops in 2000, in 2008, and in 2013.