5.2 Exponential Functions and Graphs

- Graph exponential equations and exponential functions.
- Solve applied problems involving exponential functions and their graphs.

EXPONENTIAL FUNCTION

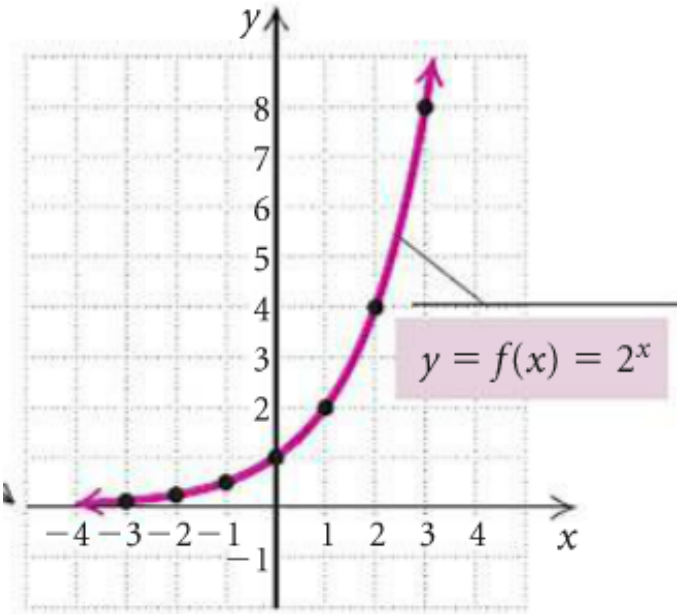
The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called the **exponential function, base a.**

EXAMPLE $y = f(x) = 2^x$

$f(0) = 2^0 = 1;$
$f(1) = 2^1 = 2;$
$f(2) = 2^2 = 4;$
$f(3) = 2^3 = 8;$

$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2};$
$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4};$
$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$

x	y $y = f(x) = 2^x$	(x, y)
0	1	(0, 1)
1	2	(1, 2)
2	4	(2, 4)
3	8	(3, 8)
−1	$\frac{1}{2}$	$(-1, \frac{1}{2})$
−2	$\frac{1}{4}$	$(-2, \frac{1}{4})$
−3	$\frac{1}{8}$	$(-3, \frac{1}{8})$



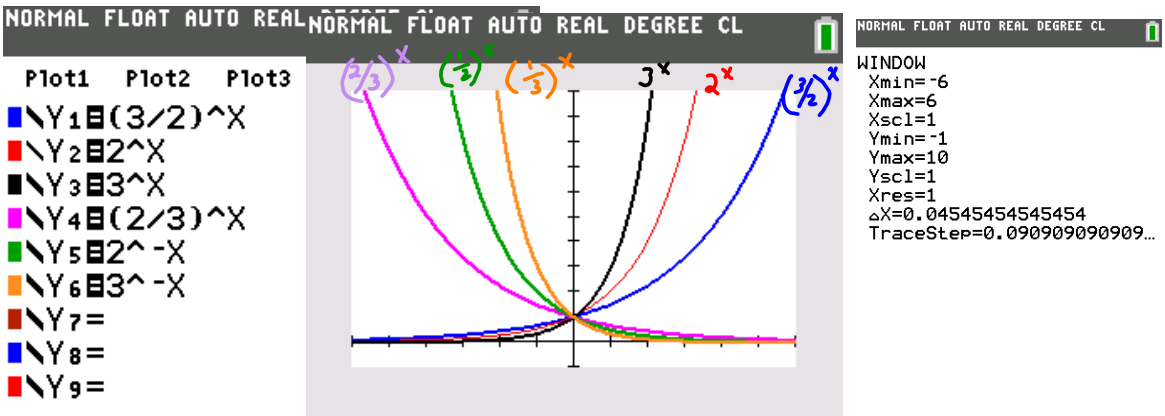
EXAMPLE $y = g(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$

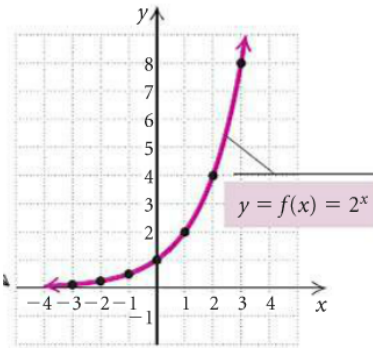
$f(x) = 2^x$

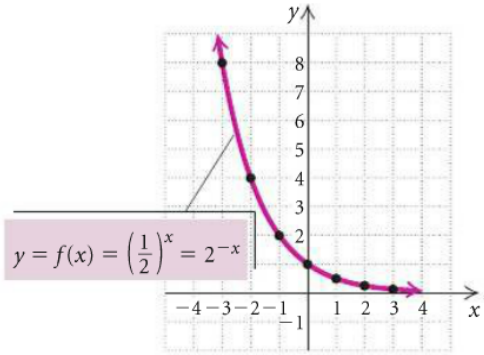
x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$g(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8







Properties of exponential functions of the form $y=a^x$

(1) y-intercept at (0,1)

(2) Domain is all real numbers, Range is all positive real numbers.

(3) The x-axis is a horizontal asymptote; never crosses x-axis.

(4) If a > 1, then it is an increasing function

(5) If 0 < a < 1, then it is a decreasing function

(6) It is a one-to-one function. It's inverse function is a logarithm.

Most important number in the universe

Most famous exponential function is

Named after Swiss mathematician. f Leonhard Euler. (1700s)

$f(x) = e^x$, where e is the euler number (pronounced "oiler")

$$e = \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k \approx 2.718281828459$$

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(1+1/100)^100
     2.704813829
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(1+1/1000)^1000
     2.716923932
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(1+1/1000000)^10
00000
     2.718280469
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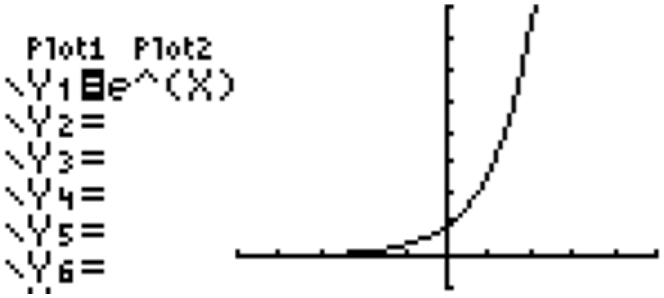

Find each of the following, to four decimal places, using a calculator.

1. e^4

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NORMAL FLOAT AUTO REAL DEGREE CL
e^(4)
                    54.59815003
```

54.5982

3. $e^{-2.458}$

```
NORMAL FLOAT AUTO REAL DEGREE CL
e^(-2.458)
                    0.0856059919
```

0.0856

Simple versus Compounded Interest

$I = Prt$; $F = P + Prt = P(1+rt)$

$100 investment at 10% ($P=100$, $r=.1$)

Simple (F)		Compounded Annually	
Year 0	100	Year 0	100
Year 1	110	Year 1	110
Year 2	120	Year 2	121
Year 3	130	Year 3	133.10
⋮	⋮	⋮	⋮
Year 50	$100 + 100(.1)(50)$ $100 + 500 = 600$	Year 50	$11,739.09

Year 0: 100

Year 1: $100 + 100(.1) = 100(1+.1) = 100(1.1) = 110$

Year 2: $110 + 110(.1) = 100(1.1) + 100(1.1)(.1)$

$= 100(1.1)(1.1) = 100(1.1)^2 = 121$

Year 3: $121 + 121(.1) = 100(1.1)^2 + 100(1.1)^2(.1)$

$= 100(1.1)^2(1.1)$

$= 100(1.1)^3$

⋮

Year n: $100(1.1)^n$

Year 50: $100(1.1)^{50}$