SECTION 5.3 *Logarithmic Functions and Graphs*

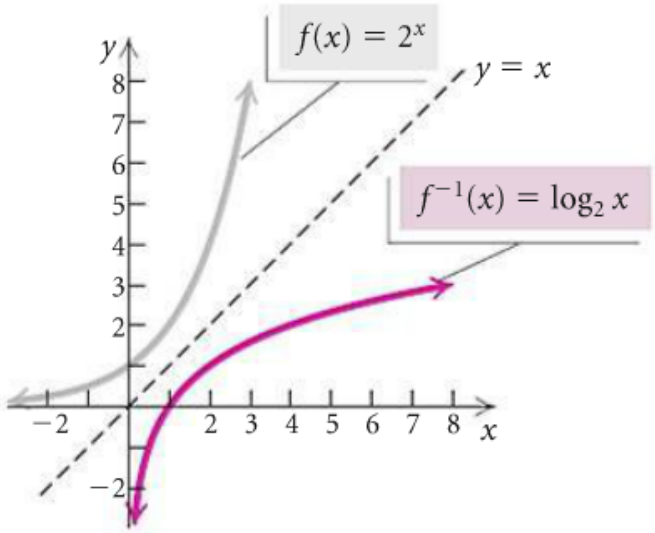
We now consider *logarithmic*, or *logarithm*, *functions*. These functions are inverses of exponential functions and have many applications.

EXAMPLE find a formula for f^{-1} when $f(x) = 2^x$

$f(x) = 2^x$	
0	1
1	2
2	4
3	8
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$
-3	$\frac{1}{8}$

$x = 2^y$	$y = \log_2 x$
1	0
2	1
4	2
8	3
$\frac{1}{2}$	-1
$\frac{1}{4}$	-2
$\frac{1}{8}$	-3

$\log_2 8 = 3$



The inverse of $f(x) = a^x$ is given by $f^{-1}(x) = \log_a x$.

LOGARITHMIC FUNCTION, BASE *a*

We define $y = \log_a x$ as that number y such that $x = a^y$, where $x > 0$ and a is a positive constant other than 1.

Find each of the following. Do not use a calculator.

10. $\log_3 9 = x = 2$

$3^x = 9$

$3^x = 3^2$

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11. $\log_5 125 = x = 3$

$5^x = 125$

$5^x = 5^3$

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12. $\log_2 64 = x = 6$

$2^x = 64$

$2^x = 2^6$

A) $\log_{27}\left(\frac{1}{9}\right) = x$

$= -2/3$

$27^x = \frac{1}{9}$

$(3^3)^x = \frac{1}{3^2}$

$3^{3x} = 3^{-2}$

$3x = -2$

$x = -2/3$

Finding Logarithms on a Calculator

$\log x$ means $\log_{10} x$. common logarithm

```
log(100)
                    2
log(.0001)
                   -4
```

```
log(89)
          1.949390007
10^1.949390007
          89.00000007
```

$\ln x$ means $\log_e x$. natural logarithm

```
ln(e^8)
                    8
ln(e)
                    1
ln(1)
                    0
```

```
ln(89)
          4.48863637
e^4.48863637
          89.00000002
```

Converting Between Exponential Equations and Logarithmic Equations

$\log_a x = y \longleftrightarrow x = a^y$

Convert to a logarithmic equation.

35. $10^3 = 1000$ $\Leftrightarrow$ $\log_{10} 1000 = 3$ $\Leftrightarrow$ $\log 1000 = 3$

37. $8^{1/3} = 2$ $\Leftrightarrow$ $\log_8 2 = \frac{1}{3}$

41. $e^2 = 7.3891$ $\Leftrightarrow$ $\log_e 7.3891 = 2$ $\Leftrightarrow$ $\ln 7.3891 = 2$

Convert to an exponential equation.

48. $\log 7 = 0.845$ $\Leftrightarrow$ $10^{0.845} = 7$

50. $\ln 0.38 = -0.9676$ $\Leftrightarrow$ $e^{-0.9676} = 0.38$

52. $\log_t Q = k$ $\Leftrightarrow$ $t^k = Q$

$\log_{\sqrt{\pi}}(t+6) = \frac{w}{2} \Leftrightarrow$

$\sqrt{\pi}^{w/2} = t + 6$

EXAMPLE Find $\log_5 8$ using common logarithms.

THE CHANGE-OF-BASE FORMULA

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

$\log_5 8$

$\frac{\log(8)}{\log(5)}$ 1.292029674

5^{Ans} 8

$\frac{\ln(8)}{\ln(5)}$ 1.292029674

5^{Ans} 8

27. $\log_{49} 7 = \frac{\ln 7}{\ln 49} = \frac{1}{2}$

NORMAL FLOAT AUTO REAL DEGREE CL

ln(20)/ln(3)

2.726833028

70. $\log_3 20 \approx 2.7268$

NORMAL FLOAT AUTO REAL DEGREE CL

$10^2 = 100$

2/log(π)

4.022931735

72. $\log_\pi 100$

4.0229

$\frac{\log(100)}{\log(\pi)}$

NORMAL FLOAT AUTO REAL DEGREE CL

ln(100)/ln(9)

2.095903274

2/log(9)

2.095903274

78. $\log_9 100$

2.0959

Graph the function and its inverse using the same set of axes. Use any method.

80. $f(x) = \log_4 x,\ f^{-1}(x) = 4^x$