5.4 Properties of Logarithmic Functions

Exponent Rule

$a^x a^y = a^{x+y}$

THE PRODUCT RULE

For any positive numbers M and N and any logarithmic base a,

$$\log_a MN = \log_a M + \log_a N.$$

(The logarithm of a product is the sum of the logarithms of the factors.)

Ex. $\log_2 (2 \cdot 4) = \log_2 2 + \log_2 4$

$\sqrt{}$: $\log_2 8 = 3$; $1 + 2 = 3$

Express as a sum of logarithms.

4. $\log_4 (64 \cdot 4) =$

$\log_4 64 + \log_4 4$

$\log_4 256 = 4$

8. $\ln ab$

$\ln a + \ln b$

THE QUOTIENT RULE

For any positive numbers M and N, and any logarithmic base a,

$$\log_a \frac{M}{N} = \log_a M - \log_a N.$$

(The logarithm of a quotient is the logarithm of the numerator minus the logarithm of the denominator.)

$\frac{a^n}{a^m} = a^{n-m}$

Express as a difference of logarithms.

18. $\log_a \frac{76}{13} = \log_a 76 - \log_a 13$

21. $\ln \frac{r}{s} = \ln r - \ln s$

☆) $\log_2 \frac{1}{8} = \log_2 1 - \log_2 8$

$= 0 - 3 = -3$

THE POWER RULE

For any positive number M, any logarithmic base a, and any real number p,

$$\log_a M^p = p \log_a M.$$

(The logarithm of a power of M is the exponent times the logarithm of M.)

$(a^n)^m = a^{nm}$

$a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Express as a product.

12. $\ln y^5 = 5 \ln y$

15. $\ln \sqrt[3]{4} = \ln 4^{\frac{1}{3}} = \frac{1}{3} \ln 4$

16. $\ln \sqrt{a} = \ln a^{\frac{1}{2}} = \frac{1}{2} \ln a$

Express in terms of sums and differences of logarithms.

$\log \dfrac{ab^2t^4}{ck^3}$ $= \log a + \log b^2 + \log t^4 - \log c - \log k^3$

$= \log a + 2\log b + 4\log t - \log c - 3\log k$

quasi-cancellation

26. $\log_b \dfrac{x^2y}{b^3}$ $= \log_b x^2 + \log_b y - \log_b b^3$

$= 2\log_b x + \log_b y - 3$

30. $\ln \sqrt[3]{5x^5}$ $= \ln (5x^5)^{1/3}$

$= \ln\left(5^{\frac{1}{3}} \cdot x^{5/3}\right)$

$= \ln 5^{\frac{1}{3}} + \ln x^{\frac{5}{3}}$

$= \frac{1}{3}\ln 5 + \frac{5}{3}\ln x$

32. $\log_c \sqrt[3]{\dfrac{y^3 z^2}{x^4}}$

$= \log_c \left(\dfrac{y^3 z^2}{x^4}\right)^{\frac{1}{3}}$

$= \log_c \dfrac{y\, z^{2/3}}{x^{4/3}}$

$= \log_c y + \log_c z^{2/3} - \log_c x^{4/3}$

$= \log_c y + \frac{2}{3}\log_c z - \frac{4}{3}\log_c x$

34. $\log_a \sqrt{\dfrac{a^6 b^8}{a^2 b^5}}$

$= \log_a \sqrt{a^4 b^3} = \log_a (a^4 b^3)^{\frac{1}{2}}$

$= \log_a a^2 b^{3/2}$

$= \log_a a^2 + \log_a b^{3/2}$

$= 2 + \frac{3}{2}\log_a b$

Express as a single logarithm and, if possible, simplify.

$\ln x - 3\ln y + \dfrac{1}{2}\ln w - 5\ln k - \dfrac{1}{3}\ln B + \ln A$

$= \ln \dfrac{x\, w^{\frac{1}{2}} A}{y^3 k^5 B^{1/3}} =$

$\ln \dfrac{A\sqrt{w}\, x}{\sqrt[3]{B}\, k^5 y^3}$