5.5

③ 5.4 $\log_a M^p = p \log_a M$

Solving Exponential Equations and Logarithmic Equations

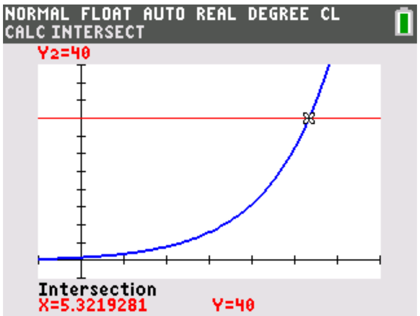
Solve the exponential equation algebraically. Then check using a graphing calculator.

6. $2^x = 40$

```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(40)/ln(2)
                    5.321928095
```

$\ln 2^x = \ln 40$

$\frac{x \cdot \ln 2}{\ln 2} = \frac{\ln 40}{\ln 2} \approx 5.3219$



14. $15^x = 30$

```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(30)/ln(15)
                    1.255958025
```

$\ln 15^x = \ln 30$

$x \ln 15 = \ln 30$

$x = \frac{\ln 30}{\ln 15} \approx 1.260$

20. $\underbrace{1000e^{0.09t}}_{\text{isolate}} = 5000$; $\div 1000$

$e^{0.09t} = 5$

$\ln e^{0.09t} = \ln 5$

$.09t = \ln 5$

$t = \dfrac{\ln 5}{.09} \approx 17.8826$

```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(5)/.09
                    17.88264347
```

$7 \cdot \underbrace{3^{2x-9}}_{\text{isolate}} + 17 = 409$

$\quad\quad\quad -17 \quad -17$

$\dfrac{7 \cdot 3^{2x-9}}{7} = \dfrac{392}{7}$

$\ln(3^{2x-9}) = \ln(56)$

$(2x-9)\ln 3 = \ln 56$

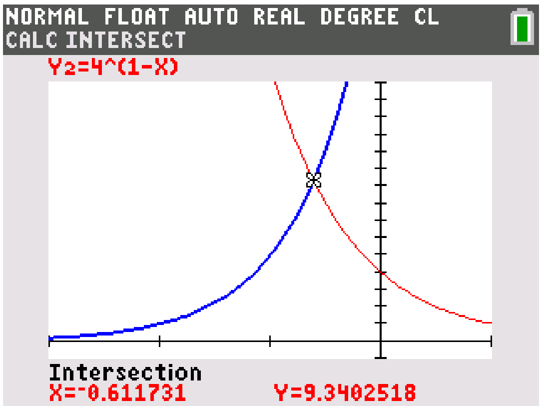
$2x-9 = \dfrac{\ln 56}{\ln 3}$

$2x-9 = 3.6640$

$\quad +9 \quad +9$

$2x = 12.6640 \Rightarrow x = 6.3320$

22. $5^{x+2} = 4^{1-x}$



$$\ln 5^{x+2} = \ln 4^{1-x}$$

$$(x+2)\ln 5 = (1-x)\ln 4$$

$$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$$

$$x\ln 5 + x\ln 4 = \ln 4 - 2\ln 5$$

$$x\cdot(\ln 5 + \ln 4) = \ln 4 - 2\ln 5$$

$$x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}$$

NORMAL FLOAT AUTO REAL DEGREE CL

(ln(4)-2ln(5))/(ln(5)+ln(4
))

⁻.611730721

$$= \frac{\ln 4 - \ln 5^2}{\ln(5\cdot 4)} = \frac{\ln 4 - \ln 25}{\ln 20} = \frac{\ln \frac{4}{25}}{\ln 20}$$

$$= \frac{\ln 0.16}{\ln 20}$$

NORMAL FLOAT AUTO REAL DEGREE CL

ln(.16)/ln(20)

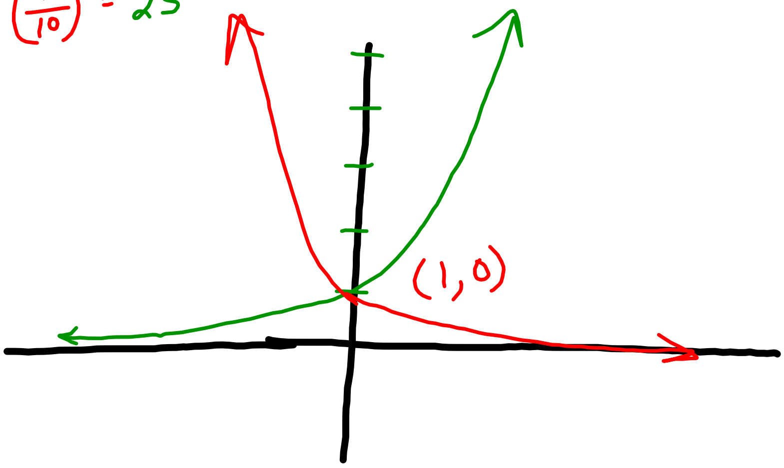
⁻.611730721

13. $10^{-x} = 5^{2x} \iff \left(\frac{1}{10}\right)^x = 25^x$

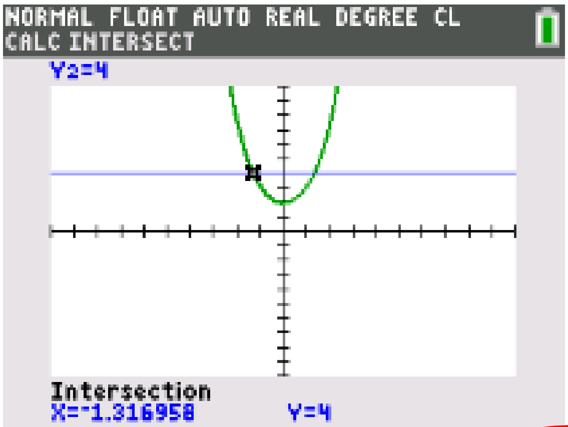
$x = 0;$

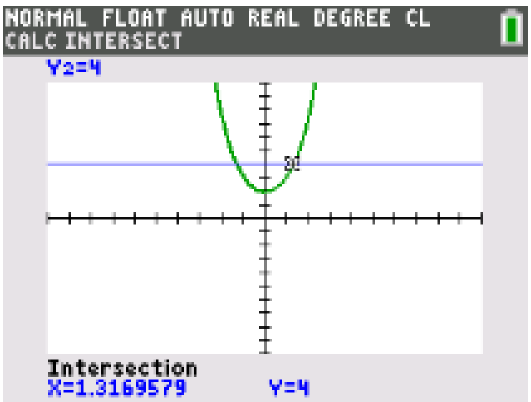
$10^0 = 5^0$

$1 = 1$



30. $e^x + e^{-x} = 4$





± 1.317

Solve the logarithmic equation algebraically. Then check using a graphing calculator.

31. $\log_5(x) = 4$

$5 \quad 5$

$x = 5^4$

$x = 625$

32. $\log_2(x) = -3$

$2 \quad 2$

$x = 2^{-3}$

$x = \frac{1}{8}$

34. $\log(x) = 1$

$10 \quad 10$

$x = 10^1$

$x = 10$

*) $\ln(x) = 0$

$e \quad e$

$x = e^0$

$x = 1$

Typical methodology to solve exponential functions:

1. Rewrite it so it is in the form $b^x = N$
2. Take the log of the base of each side: ex. $\log\left(b^x\right) = \log(N) \Leftrightarrow x \log b = \log N$
3. Solve for x

Typical methodology to solve logarithmic functions:

1. Rewrite it so that you have a single logarithm equal to a single logarithm or a number
2. Exponentiate both sides with the base of the logarithm
3. Solve for x
4. Check for extraneous roots; recall, you can only take the logarith of a positive number.

41. $\log x + \log (x-9) = 1$

$\log_a MN =$
$\log_a M + \log_a N$

① $\log_{10} [x(x-9)] = 1_{10}$

② $x(x-9) = 10$

③ $x^2 - 9x = 10$

$x^2 - 9x - 10 = 0$

$(x-10)(x+1) = 0$

④ $\cancel{-1}$, 10

e
r
r
a

40. $\log_5 (8 - 7x) = 3$

5 5

$\Rightarrow$ $8 - 7x = 125$

$-8 \quad\quad -8$

$-7x = 117$

$x = \frac{-117}{7}$

45. $\log_8(x+1) - \log_8 x = 2$

$\log_8 \frac{x+1}{x} = 2$

$8 \quad\quad 8$

$\Rightarrow x\,\frac{x+1}{x} = 64x$

$x+1 = 64x$

$64x = x+1$

$-x \quad -x$

$\frac{63x}{63} = \frac{1}{63}$ $\Rightarrow$ $x = \frac{1}{63}$

48. $\log_3(x+14) - \log_3(x+6) = \log_3 x$

$\log_3 \frac{x+14}{x+6} = \log_3 x$

$3 \quad\quad 3$

$\Rightarrow (x+6)\frac{x+14}{x+6} = x(x+6)$

$x+14 = x^2+6x$

$x^2+6x = x+14$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x = \cancel{-7}, 2$

extra

54. $\log_5(x+4) + \log_5(x-4) = 2$

$\log_5[(x+4)(x-4)] = 2$

$\log_5(x^2-16) = 2$

$5 \quad\quad 5$

$\Rightarrow x^2-16 = 25$

$\Rightarrow x^2 = 41$

$\Rightarrow x = \cancel{-\sqrt{41}}, \sqrt{41}$

extra

55. $\ln(x+8) + \ln(x-1) = 2\ln x$

$$\ln[(x+8)(x-1)] = \ln x^2$$

$$e^{\ln(x^2+7x-8)} = e^{\ln x^2}$$

$$\begin{array}{rcl} x^2+7x-8 & = & x^2 \\ -x^2 & & -x^2 \\ \hline 7x-8 & = & 0 \end{array} \Rightarrow 7x = 8$$

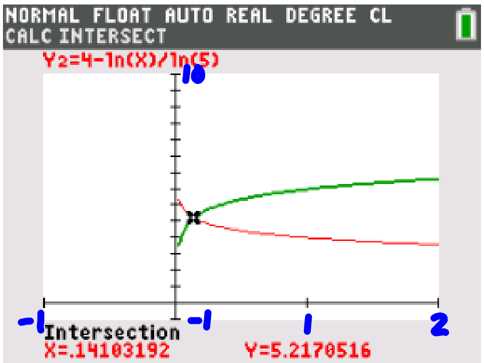
$$\Rightarrow x = 8/7$$

72. $\log_3 x + 7 = 4 - \log_5 x$

one is base 3, the other is base 5. It's best to use graphing.

$$y_1 = \frac{\ln(x)}{\ln(3)} + 7$$

$$y_2 = 4 - \frac{\ln(x)}{\ln(5)}$$



$x \approx 0.1410$