Typical methodology to solve exponential functions:

exponential = number

log of any base, I prefer ln

1. Rewrite it so it is in the form $b^x = N$
2. Take the log of the base of each side: ex. $\log\left(b^x\right) = \log\left(N\right) \Leftrightarrow x\log b = \log N$
3. Solve for x

20. $\dfrac{1000e^{0.09t}}{1000} = \dfrac{5000}{1000}$

```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(5)/.09
                    17.88264347
```

$e^{0.09t} = 5$

$\ln e^{0.09t} = \ln 5$

$0.09t = \ln 5 \Rightarrow t = \dfrac{\ln 5}{0.09} \approx 17.8826$

*) $7 \cdot 3^{2x-9} + 17 = 409$

isolate -17 -17

$\dfrac{7 \cdot 3^{2x-9}}{7} = \dfrac{392}{7}$

$3^{2x-9} = 56 \Rightarrow \ln 3^{2x-9} = \ln 56$

$(2x-9)\ln 3 = \ln 56$

$2x - 9 = \dfrac{\ln 56}{\ln 3}$

```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(56)/ln(3)
                    3.66403301
```

$2x - 9 \approx 3.664 \Rightarrow 2x = 12.664$

$\Rightarrow x = 6.332$

22. $5^{x+2} = 4^{1-x}$

$\ln 5^{x+2} = \ln 4^{1-x} \Rightarrow$

note the importance of parentheses here

$(x+2)\ln 5 = (1-x)\ln 4$

use the distributive property; ln 4 and ln 5 are just numbers:
$\ln 4 \approx 1.39$
$\ln 5 \approx 1.61$

$x\ln 5 + 2\ln 5 = \ln 4 - x\ln 4$

any term that has an "x" gets pulled to the left-hand side

$x\ln 5 + x\ln 4 = \ln 4 - 2\ln 5$; now, on the left-hand side "x" is the GCF

$x\cdot(\ln 5 + \ln 4) = \ln 4 - 2\ln 5$

$x = \dfrac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4} \approx -0.6117$

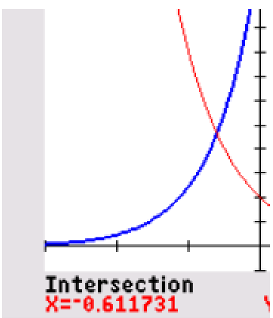
$= \dfrac{\ln 4 - \ln 5^2}{\ln(5\cdot 4)} = \dfrac{\ln \frac{4}{25}}{\ln 20} = \dfrac{\ln 0.16}{\ln 20}$

```
NORMAL FLOAT AUTO REAL DEGREE CL
(ln(4)-2ln(5))/(ln(5)+ln(4
))
                 -0.611730721
```

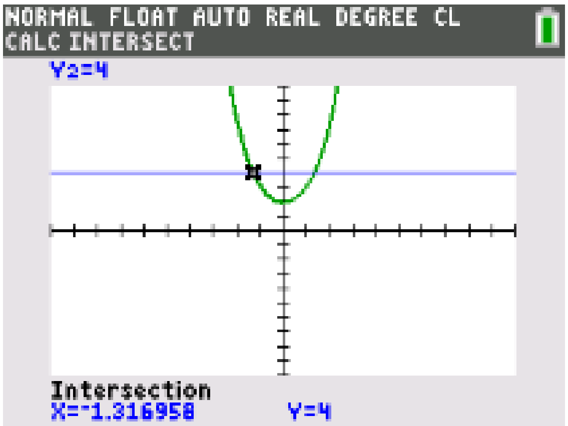
```
NORMAL FLOAT AUTO REAL DEGREE CL
ln(.16)/ln(20)
                 -0.611730721
```

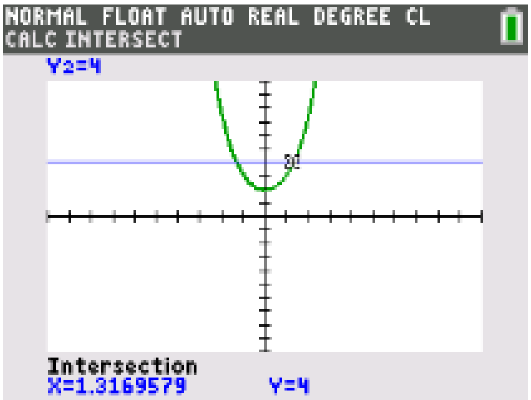
```
\Y1=5^(X+2)
\Y2=4^(1-X)

WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-2
 Ymax=20
 Yscl=2
```



30. $e^x + e^{-x} = 4$


NORMAL FLOAT AUTO REAL DEGREE CL
CALC INTERSECT
Y2=4
Intersection
X=⁻1.316958
Y=4


NORMAL FLOAT AUTO REAL DEGREE CL
CALC INTERSECT
Y2=4
Intersection
X=1.3169579
Y=4

Solve the logarithmic equation algebraically. Then check using a graphing calculator.

quasi - cancellation

① $\log_b b^x = x$

② $b^{\log_b x} = x$

31. $\log_5 x = 4$

5 5

$x = 5^4 = 625$

32. $\log_2 x = -3$

2 2

$\Rightarrow x = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

34. $\log x = 1$

10 10

$\Rightarrow x = 10^1$

$x = 10$

✱) $\ln x = 0$

e e

$\Rightarrow x = e^0$

$x = 1$

Typical methodology to solve logarithmic functions:

1. Rewrite it so that you have a single logarithm equal to a single logarithm or a number
2. Exponentiate both sides with the base of the logarithm
3. Solve for x
4. Check for extraneous roots; recall, you can only take the logarith of a positive number.

40. $\log_5(8-7x) = 3$

5 5

$\Rightarrow$ $8 - 7x = 125$

$-8 \qquad -8$

$-7x = 117$

$x = \frac{-117}{7}$

note: $8-7\left(\frac{-117}{7}\right)$ is positive;
thus, x is not extraneous

41. $\log x + \log(x-9) = 1$

*) $\log_b MN = \log_b M + \log_b N$

$\log[x(x-9)] = 1$

10 10

$x(x-9) = 10 \Rightarrow$

$x^2 - 9x - 10 = 0 \Rightarrow$

$(x-10)(x+1) = 0 \Rightarrow$

$x - 10 = 0$ | $x + 1 = 0$

$x = 10$ ✓ | $x = -1$ extraneous

note: $\log(-1)$ dne

45. $\log_8(x+1) - \log_8 x = 2$

$\log_b \frac{M}{N} = \log_b M - \log_b N$

$8^{\log_8\left(\frac{x+1}{x}\right)} = 8^2$

$x \cdot \frac{x+1}{x} = 64 \cdot x \Rightarrow x+1 = 64x$

$\Rightarrow 1 = 63x$

$\Rightarrow x = \frac{1}{63}$ ✓

48. $\log_3(x+14) - \log_3(x+6) = \log_3 x$

$3^{\log_3 \frac{x+14}{x+6}} = 3^{\log_3 x}$

$(x+6)\frac{x+14}{x+6} = x(x+6) \Rightarrow x+14 = x^2+6x$

$\Rightarrow x^2+5x-14 = 0$

$\Rightarrow (x+7)(x-2) = 0$

$\Rightarrow x = \cancel{-7}$, $x = 2$ ✓

e
f
a

54. $\log_5(x+4) + \log_5(x-4) = 2$

$5^{\log_5[(x+4)(x-4)]} = 5^2$

$(x+4)(x-4) = 25$

$x^2 - 16 = 25$

$x^2 = 41 \Rightarrow x = \cancel{-\sqrt{41}}, \sqrt{41}$ ✓

e
f
a

55. $\ln(x+8) + \ln(x-1) = 2\ln x$

$$e^{\ln[(x+8)(x-1)]} = e^{\ln x^2}$$

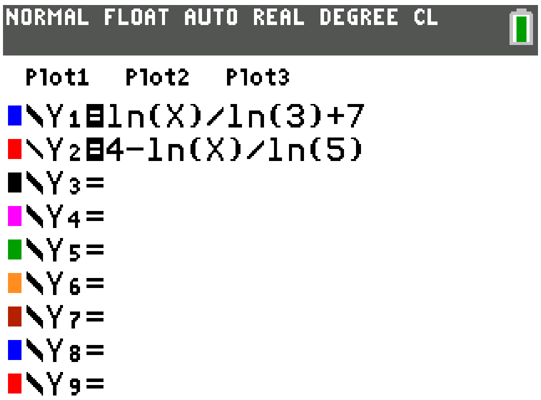
$(x+8)(x-1) = x^2 \Rightarrow$

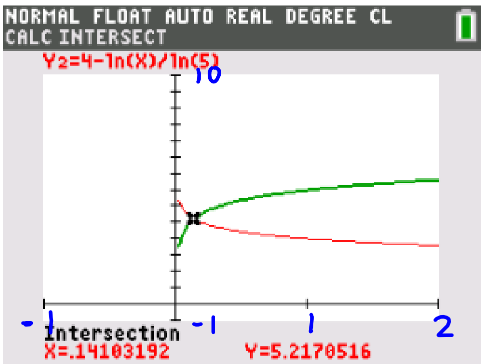
$x^2 + 7x - 8 = x^2 \Rightarrow$

$7x - 8 = 0 \Rightarrow$

$7x = 8 \Rightarrow x = \frac{8}{7}$ ✓

72. $\underbrace{\log_3 x + 7}_{y_1} = \underbrace{4 - \log_5 x}_{y_2}$





$x \approx 0.1410$