Chapter 11: Analyzing Association Between Quantitative Variables: Regression Analysis

Section 11.5: Exponential Regression: A Model for Nonlinearity

1

Learning Objectives

1. Nonlinear Regression Models
2. Exponential Regression Model
3. Interpreting Exponential Regression Models

2

Learning Objective 1: Nonlinear Regression Models

- **If a scatterplot indicates substantial curvature in a relationship, then equations that provide curvature are needed**
  - **Occasionally a scatterplot has a parabolic appearance: as x increases, y increases then it goes back down**
  - **More often, y tends to continually increase or continually decrease but the trend shows curvature**

3

Learning Objective 1: Example: Exponential Growth in Population Size

- **Since 2000, the population of the U.S. has been growing at a rate of 2% a year**
  - **The population size in 2000 was 280 million**
  - **The population size in 2001 was 280 x 1.02**
  - **The population size in 2002 was 280 x $(1.02)^2$**
  - **…**
  - **The population size in 2010 is estimated to be**
  - **280 x $(1.02)^{10}$**
  - **This is called *exponential growth***

4

Learning Objective 2: Exponential Regression Model

- An exponential regression model has the formula:

$$\mu_y = \alpha\beta^x$$

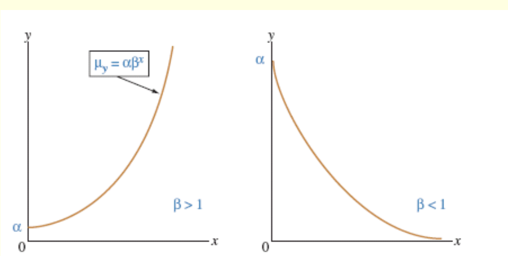
For the mean μ_y of y at a given value of x, where α and β are parameters

Learning Objective 2: Exponential Regression Model

- In the exponential regression equation, the explanatory variable x appears as the exponent of a parameter
- The mean μ_y and the parameter β can take only positive values

Learning Objective 2: Exponential Regression Model

- As x increases, the mean μ_y increases when β>1
- It continually decreases when 0 < β<1



Learning Objective 2: Exponential Regression Model

- For exponential regression, the ***logarithm of the mean*** is a linear function of x
- When the exponential regression model holds, a plot of the log of the y values versus x should show an approximate straight-line relation with x

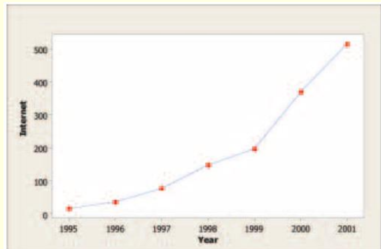
Learning Objective 2:
Example: Explosion in Number of People Using the Internet

TABLE 12.9: Number of People Worldwide Using Internet (in Millions)

Year	No. Years Since 1995 x	Number People y	Log No. People $log(y)$	Predicted Number $\hat{y}$
1995	0	16	1.20	20.4
1996	1	36	1.56	36.1
1997	2	76	1.88	63.9
1998	3	147	2.17	113.2
1999	4	195	2.29	200.4
2000	5	369	2.57	354.9
2001	6	513	2.71	628.4

Learning Objective 2:
Example: Explosion in Number of People Using the Internet

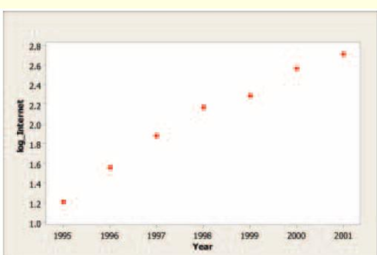
Plot of Number of People Using Internet between 1995 and 2001



10

Learning Objective 2:
Example: Explosion in Number of People Using the Internet

Plot of **Log** Number of People Using Internet between 1995 and 2001



11

Learning Objective 2:
Example: Explosion in Number of People Using the Internet

- **Using regression software, we can create the exponential regression equation:**
  - **x: the number of years since 1995. Start with x = 0 for 1995, then x=1 for 1996, etc**
  - **y: number of internet users**
  - **Equation:**

$$\hat{y} = 20.38(1.7708)^x$$

12

Learning Objective 3:
Interpreting Exponential Regression Models

- **In the exponential regression model,**

$$\mu_y = \alpha\beta^x$$

- **the parameter α represents the mean value of y when x = 0;**
- **The parameter β represents the multiplicative effect on the mean of y for a one-unit increase in x**

13

Learning Objective 3:
Example: Explosion in Number of People Using the Internet

- **In this model:**

$$\hat{y} = 20.38(1.7708)^x$$

- **The predicted number of Internet users in 1995 (for which x = 0) is 20.38 million**
- **The predicted number of Internet users in 1996 is 20.38 *times* 1.7708**

14