Chapter 8: Statistical Inference: Significance Tests about Hypotheses

Section 8.3: Significance Tests About Means

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Learning Objectives

1. Steps of a Significance Test about a Population Mean
2. Summary of P-values for Different Alternative Hypotheses
3. Example: Significance Test for a Population Mean
4. Results of Two-Sided Tests and Results of Confidence Intervals Agree
5. What If the Population Does Not Satisfy the Normality Assumption?
6. Regardless of Robustness, Look at the Data

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Learning Objective 1: Steps of a Significance Test About a Population Mean

- ***Step 1: Assumptions***
  - **The variable is quantitative**
  - **The data are obtained using randomization**
  - **The population distribution is approximately normal. This is most crucial when *n* is small and H_a is one-sided.**

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Learning Objective 1: Steps of a Significance Test About a Population Mean

- ***Step 2: Hypotheses:***
- **The null hypothesis has the form**:
  - **H_0: $\mu = \mu_0$**
- **The alternative hypothesis has the form**:
  - **H_a: $\mu > \mu_0$ (one-sided test) or**
  - **H_a: $\mu < \mu_0$ (one-sided test) or**
  - **H_a: $\mu \neq \mu_0$ (two-sided test)**

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Learning Objective 1: Steps of a Significance Test About a Population Mean

- ***Step 3: Test Statistic***
  - **The test statistic measures how far the sample mean falls from the null hypothesis value μ_0, as measured by the number of standard errors between them**
  - **The test statistic is:**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

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Learning Objective 1: Steps of a Significance Test About a Population Mean

- ***Step 4: P-value***
  - **The P-value summarizes the evidence**
  - **It describes how unusual the data would be if H_0 were true**

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Learning Objective 1:
Steps of a Significance Test About a Population Mean

- ***Step 5: Conclusion***
  - **We summarize the test by reporting and interpreting the P-value**

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Learning Objective 2:
Summary of P-values for Different Alternative Hypotheses

Alternative Hypothesis	P-value
$H_a: \mu > \mu_0$	Right-tail probability from t distribution
$H_a: \mu < \mu_0$	Left-tail probability from t distribution
$H_a: \mu \neq \mu_0$	Two-tail probability from t distribution

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **A study compared different psychological therapies for teenage girls suffering from anorexia**
- **The variable of interest was each girl's weight change: 'weight at the end of the study' – 'weight at the beginning of the study'**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **One of the therapies was cognitive therapy**
- **In this study, 29 girls received the therapeutic treatment**
- **The weight changes for the 29 girls had a sample mean of 3.00 pounds and standard deviation of 7.32 pounds**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

TABLE 8.3: Weights of Anorexic Girls (in Pounds) before and after Treatment

Example 7 uses the weight change as the variable of interest.

	Weight				Weight				Weight			
Girl	Before	After	Change	Girl	Before	After	Change	Girl	Before	After	Change
1	80.5	82.2	1.7	11	85.0	96.7	11.7	21	83.0	81.6	−1.4
2	84.9	85.6	0.7	12	89.2	95.3	6.1	22	76.5	75.7	−0.8
3	81.5	81.4	−0.1	13	81.3	82.4	1.1	23	80.2	82.6	2.4
4	82.6	81.9	−0.7	14	76.5	72.5	−4.0	24	87.8	100.4	12.6
5	79.9	76.4	−3.5	15	70.0	90.9	20.9	25	83.3	85.2	1.9
6	88.7	103.6	14.9	16	80.6	71.3	−9.3	26	79.7	83.6	3.9
7	94.9	98.4	3.5	17	83.3	85.4	2.1	27	84.5	84.6	0.1
8	76.3	93.4	17.1	18	87.7	89.1	1.4	28	80.8	96.2	15.4
9	81.0	73.4	−7.6	19	84.2	83.9	−0.3	29	87.4	86.7	−0.7
10	80.5	82.1	1.6	20	86.4	82.7	−3.7				

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **How can we frame this investigation in the context of a significance test that can detect whether the therapy was effective?**
- **Null hypothesis: "no effect"**
- **Alternative hypothesis: therapy is "effective"**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **Step 1: Assumptions**
  - **The variable (weight change) is quantitative**
  - **The subjects were a *convenience sample*, rather than a random sample. The question is whether these girls are a good representation of all girls with anorexia.**
  - **The population distribution is approximately normal**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **Step 2: Hypotheses**
  - **H_0: μ = 0**
  - **H_a: μ > 0**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **Step 3: Test Statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{(3.00 - 0)}{7.32/\sqrt{29}} = 2.21$$

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **Step 4: P-value**

  The P-value is the area to the right of t=2.21 for the t sampling distribution with 28 df. This values is 0.018.
  - **If the treatment had no effect, the probability of obtaining a sample this extreme would be 0.018**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **Step 5: Conclusion**
  - **The small P-value of 0.018 provides considerable evidence against the null hypothesis (the hypothesis that the therapy had no effect)**

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Learning Objective 3:
Example: Mean Weight Change in Anorexic Girls

- **"The diet had a statistically significant positive effect on weight (mean change = 3 pounds, n = 29, t = 2.21, P-value = 0.018)"**
- **The effect, however, may be small in practical terms**
  - **95% CI for μ: (0.2, 5.8) pounds**

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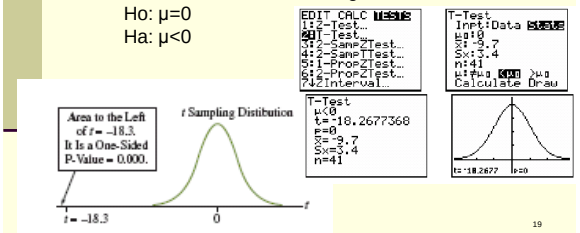
Learning Objective 3:
Example: Does Low Carb Diet Work?

- After 16 weeks on a diet, 41 subjects lost an average of 9.7 kg with a standard deviation of 3.4 kg
- Calculate the P-value for testing:
  - Ho: μ=0
  - Ha: μ<0



Learning Objective 4:
Results of Two-Sided Tests and Results of Confidence Intervals Agree

- **Conclusions about means using two-sided significance tests are consistent with conclusions using confidence intervals**
  - **If P-value ≤ 0.05 in a two-sided test, a 95% confidence interval does not contain the value specified by the null hypothesis**
  - **If P-value > 0.05 in a two-sided test, a 95% confidence interval does contain the value specified by the null hypothesis**

Learning Objective 5:
What If the Population Does Not Satisfy the Normality Assumption?

- **For large samples (roughly about 30 or more) this assumption is usually not important**
  - **The sampling distribution of $\bar{x}$ is approximately normal regardless of the population distribution**

Learning Objective 5:
What If the Population Does Not Satisfy the Normality Assumption?

- **In the case of small samples, we cannot assume that the sampling distribution of $\bar{x}$ is approximately normal**
  - **Two-sided inferences using the *t* distribution are robust against violations of the normal population assumption. They still usually work well if the actual population distribution is not normal**
  - **The test does not work well for a one-sided test with small *n* when the population distribution is highly skewed**

Learning Objective 6:
Regardless of Robustness, Look at the Data

- **Whether *n* is small or large, you should look at the data to check for severe skew or for severe outliers**
  - **In these cases, the sample mean could be a misleading measure**