Chapter 8:Statistical Inference: Significance Tests about Hypothesis

Section 8.6: How Likely is a Type II Error

Learning Objectives

1. Type II Error
2. Calculating Type II Error
3. Power of a Test

Learning Objective 1:
Type II Error

- **A Type II error occurs in a hypothesis test when we fail to reject H_0 even though it is actually false**
- **To calculate the probability of a Type II error, we must perform separate calculations for various values of the parameter of interest**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **Scientific "test of astrology" experiment:**
  - **For each of 116 adult volunteers, an astrologer prepared a horoscope based on the positions of the planets and the moon at the moment of the person's birth**
  - **Each adult subject also filled out a California Personality Index (CPI) Survey**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **For a given adult, his or her birth data and horoscope were shown to the astrologer together with the results of the personality survey for that adult and for two other adults randomly selected from the group**
- **The astrologer was asked which personality chart of the 3 subjects was the correct one for that adult, based on his or her horoscope**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **28 astrologers were randomly chosen to take part in the experiment**
- **The National Council for Geocosmic Research claimed that the probability of a correct guess on any given trial in the experiment was larger than 1/3, the value for random guessing**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **With random guessing, $p = 1/3$**
- **The astrologers' claim: $p > 1/3$**
- **The hypotheses for this test:**
  - **H_o: $p = 1/3$**
  - **H_a: $p > 1/3$**
- **The significance level used for the test is 0.05**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **For what values of the sample proportion can we reject H_0?**
- **A test statistic of $z = 1.645$ has a P-value of 0.05. So, we reject H_0 for $z \geq 1.645$ and we fail to reject H_0 for $z < 1.645$.**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **Find the value of the sample proportion that would give us a z of 1.645:**

$$z = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0(1-p_0)}{n}}}, \text{solving for } \hat{p}:$$

$$\hat{p} = 1/3 + 1.645\sqrt{\frac{(1/3)(2/3)}{116}} = 0.405$$

Learning Objective 2:
Example 1: Calculating a Type II Error

- **So, we fail to reject H_0 if** $\hat{p} < 0.405$
- **Suppose that in reality astrologers can make the correct prediction 50% of the time (that is, p = 0.50)**
- **In this case, (p = 0.50), we can now calculate the probability of a Type II error**

Learning Objective 2:
Example 1: Calculating a Type II Error

- **We calculate the probability of a sample proportion < 0.405 assuming that the true proportion is 0.50**

$$z = \frac{0.405 - 0.50}{\sqrt{\dfrac{0.50(1 - 0.50)}{116}}} = -2.04$$

Learning Objective 2:
Example 1: Calculating a Type II Error

- **The area to the left of -2.04 in the standard normal table is 0.02**
- **The probability of making a Type II error and failing to reject H_0: p = 1/3 is only 0.02 in the case for which the true proportion is 0.50**
- **This is only a small chance of making a Type II error**

Learning Objective 1:
Type II Error

For a fixed significance level α, P(Type II error) decreases

- as the parameter value moves farther into the H_a values and away from the H_0 value
- as the sample size increases

Learning Objective 3: Power of a Test

- **Power = 1 – P(Type II error)=probability of rejecting the null hypothesis when it is false**

- **The higher the power, the better**

- **In practice, it is ideal for studies to have high power while using a relatively small significance level**

Learning Objective 3:
Power of a Test

- **In this example, the Power of the test when $p = 0.50$ is: $1 – 0.02 = 0.98$**
- **Since, the higher the power the better, a test with power of 0.98 is quite good**