Chapter 9: Comparing Two Groups

Section9.2: Quantitative Response: How Can We Compare Two Means?

Learning Objectives

1. Comparing Means
2. Standard Error for Comparing Two Means
3. Confidence Interval for the Difference between Two Population Means
4. Example: Nicotine – How Much More Addicted Are Smokers than Ex-Smokers?
5. How Can We Interpret a Confidence Interval for a Difference of Means?
6. Significance Tests Comparing Population Means

Learning Objective 1: Comparing Means

- We can compare two groups on a ***quantitative response variable*** by comparing their means

Learning Objective 1: Example: Teenagers Hooked on Nicotine

- **A 30-month study:**
  - **Evaluated the degree of addiction that teenagers form to nicotine**
  - **332 students who had used nicotine were evaluated**
  - **The response variable was constructed using a questionnaire called the Hooked on Nicotine Checklist (HONC)**

Learning Objective 1: Example: Teenagers Hooked on Nicotine

- **The HONC score is the total number of questions to which a student answered "yes" during the study**
- **The higher the score, the more hooked on nicotine a student is judged to be**

Learning Objective 1: Example: Teenagers Hooked on Nicotine

- **The study considered explanatory variables, such as gender, that might be associated with the HONC score**

TABLE 9.5: Summary of Hooked on Nicotine Checklist (HONC) Scores, by Gender

		HONC Score	
Group	Sample Size	Mean	Standard Deviation
Females	150	2.8	3.6
Males	182	1.6	2.9

Learning Objective 1:
Example: Teenagers Hooked on Nicotine

- **How can we compare the sample HONC scores for females and males?**
- **We estimate (μ_1 - μ_2) by ($\bar{x}_1 - \bar{x}_2$):**

  2.8 – 1.6 = 1.2

- **On average, females answered "yes" to about one more question on the HONC scale than males did**

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Learning Objective 1:
Example: Teenagers Hooked on Nicotine

- **To make an inference about the difference between population means, ($\mu_1 - \mu_2$), we need to learn about the variability of the sampling distribution of:**

$$(\bar{x}_1 - \bar{x}_2)$$

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Learning Objective 2:
Standard Error for Comparing Two Means

- **The difference, $(\bar{x}_1 - \bar{x}_2)$, is obtained from sample data. It will vary from sample to sample.**
- **This variation is the standard error of the sampling distribution of $(\bar{x}_1 - \bar{x}_2)$:**

$$se = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

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Learning Objective 3:
Confidence Interval for the Difference Between Two Population Means

A **confidence interval for $\mu_1 - \mu_2$** is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- $t_{.025}$ is the critical value for a 95% confidence level from the t distribution
- The **degrees of freedom** are calculated using software. If you are not using software, you can take *df* to be the smaller of (n_1-1) and (n_2-1) as a "safe" estimate

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Learning Objective 3:
Confidence Interval for the Difference between Two Population Means

- **This method assumes**:
  - **Independent random samples from the two groups**
  - **An approximately normal population distribution for each group**
    - **this is mainly important for small sample sizes, and even then the method is robust to violations of this assumption**

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Learning Objective 4:
Example: Nicotine – How Much More Addicted Are Smokers than Ex-Smokers?

- **Data as summarized by HONC scores for the two groups:**
- **Smokers: $\bar{x}_1$ = 5.9, s_1 = 3.3, n_1 = 75**
- **Ex-smokers: $\bar{x}_2$ = 1.0, s_2 = 2.3, n_2 = 257**

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Learning Objective 4:
Example: Nicotine – How Much More Addicted Are Smokers than Ex-Smokers?

- **Were the sample data for the two groups approximately normal?**
  - **Most likely not for Group 2 (based on the sample statistics: $\bar{x}_2$= 1.0, s_2 = 2.3)**
  - **Since the sample sizes are large, this lack of normality is not a problem**

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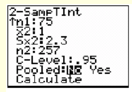
Learning Objective 4:
Example: Nicotine – How Much More Addicted Are Smokers than Ex-Smokers?

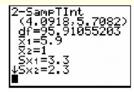
- **95% CI for (μ_1- μ_2):**

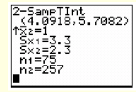
$$(5.9-1) \pm 1.985\sqrt{\frac{3.3^2}{75}+\frac{2.3^2}{257}} =$$

$$4.9 \pm 0.8, \textit{ or } (4.1, 5.7)$$

- **We can infer that the population mean for the smokers is between 4.1 higher and 5.7 higher than for the ex-smokers**







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Learning Objective 4:
Example: Exercise and Pulse Rates

A study is performed to compare the mean resting pulse rate of adult subjects who exercise regularly to the mean resting pulse rate of those who do not exercise regularly.

	n	mean	std. dev.
Exercisers	29	66	8.6
Non-exercisers	31	75	9.0

This is an example of when to use the two-sample t procedures.

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Learning Objective 4:
Example: Exercise and Pulse Rates

Find a 95% confidence interval for the difference in population means (non-exercisers minus exercisers).

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 75-66 \pm 2.048\sqrt{\frac{(9.0)^2}{31}+\frac{(8.6)^2}{29}}$$

$$= 9 \pm 4.65$$

$$= 4.35 \text{ to } 13.65$$

- Note: we use the “safe” estimate of 29-1=28 for our degrees of freedom in this calculation

“We are 95% confident that the difference in mean resting pulse rates (non-exercisers minus exercisers) is between 4.35 and 13.65 beats per minute.”

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Learning Objective 4:
Class Exercise 1

- Attitude toward mathematics was measured for two different groups. The attitude scores range from 0 to 80 with the higher scores indicating a more positive attitude. The first group consisted of Elementary education majors and the other group consisted of majors from several other areas. The results were as follows:

	N	mean	SD
Elementary Ed	75	42.7	15.5
Non Elem. Ed	110	49.3	17.0

- Find a 95% confidence interval for μ_1 - μ_2

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Learning Objective 4:
Class Exercise 2

- Are girls less inclined to enroll in science courses than boys? One recent study of fourth, fifth, and sixth graders asked how many science courses they intended to take. The resulting data were used to compute the following summary statistics:

	n	Mean	SD
Males	203	3.42	1.49
Females	224	2.42	1.35

- Calculate a 99% confidence interval for the difference between males and females in mean number of science courses planned

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Learning Objective 5: How Can We Interpret a Confidence Interval for a Difference of Means?

- **Check whether 0 falls in the interval**
- **When it does, 0 is a plausible value for $(\mu_1 - \mu_2)$, meaning that it is possible that $\mu_1 = \mu_2$**
- **A confidence interval for $(\mu_1 - \mu_2)$ that contains only positive numbers suggests that $(\mu_1 - \mu_2)$ is positive**
  - **We then infer that μ_1 is larger than μ_2**

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Learning Objective 5: How Can We Interpret a Confidence Interval for a Difference of Means?

- **A confidence interval for $(\mu_1 - \mu_2)$ that contains only negative numbers suggests that $(\mu_1 - \mu_2)$ is negative**
  - **We then infer that μ_1 is smaller than μ_2**
- **Which group is labeled '1' and which is labeled '2' is arbitrary**

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Learning Objective 6: Significance Tests Comparing Population Means

1. Assumptions:

- **Quantitative response variable for two groups**
- **Independent random samples**

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Learning Objective 6: Significance Tests Comparing Population Means

Assumptions (continued):

- **Approximately normal population distributions for each group**
  - **This is mainly important for small sample sizes, and even then the two-sided *t* test is robust to violations of this assumption**

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Learning Objective 6: Significance Tests Comparing Population Means

2. Hypotheses:

The null hypothesis is the hypothesis of *no difference* or *no effect*:

$H_0: (\mu_1 - \mu_2) = 0$

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Learning Objective 6: Significance Tests Comparing Population Proportions

2. Hypotheses (continued):

The alternative hypothesis:

$H_a: (\mu_1 - \mu_2) \neq 0$ **(two-sided test)**

$H_a: (\mu_1 - \mu_2) < 0$ **(one-sided test)**

$H_a: (\mu_1 - \mu_2) > 0$ **(one-sided test)**

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Learning Objective 6: Significance Tests Comparing Population Means

3. The test statistic is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Note change from "z" to "t" in formula

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Learning Objective 6: Significance Tests Comparing Population Means

4. P-value: Probability obtained from the standard normal table

5. Conclusion: Smaller P-values give stronger evidence against H_0 and supporting H_a

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Learning Objective 6: Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Experiment:**
  - **64 college students**
  - **32 were randomly assigned to the cell phone group**
  - **32 to the control group**

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Learning Objective 6: Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Experiment (continued):**
  - **Students used a machine that simulated driving situations**
  - **At irregular periods a target flashed red or green**
  - **Participants were instructed to press a "brake button" as soon as possible when they detected a red light**

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Learning Objective 6: Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **For each subject, the experiment analyzed their mean response time over all the trials**
- **Averaged over all trials and subjects, the *mean* response time for the cell-phone group was 585.2 milliseconds**
- **The *mean* response time for the control group was 533.7 milliseconds**

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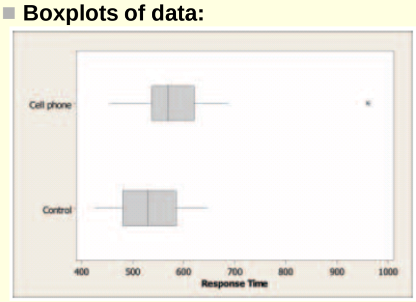
Learning Objective 6: Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Boxplots of data:**



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Learning Objective 6:
Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Test the hypotheses:**

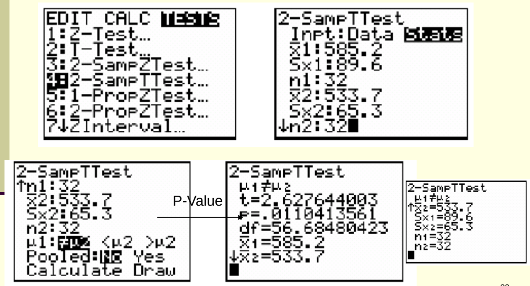
  H_0: $(\mu_1 - \mu_2) = 0$

  vs.

  H_a: $(\mu_1 - \mu_2) \neq 0$

  - **using a significance level of 0.05**

Learning Objective 6:
Example: Does Cell Phone Use While Driving Impair Reaction Times?



Learning Objective 6:
Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Conclusion:**
  - **The P-value is less than 0.05, so we can reject H_0**
  - **There is enough evidence to conclude that the population mean response times differ between the cell phone and control groups**
  - **The sample means suggest that the population mean is higher for the cell phone group**

Learning Objective 6:
Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **What do the box plots tell us?**
  - **There is an extreme outlier for the cell phone group**
  - **It is a good idea to make sure the results of the analysis aren't affected too strongly by that single observation**
    - **Delete the extreme outlier and redo the analysis**
    - **In this example, the t-statistic changes only slightly**

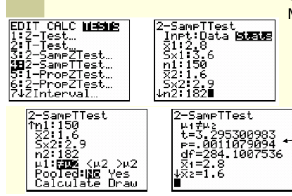
Learning Objective 6:
Example: Does Cell Phone Use While Driving Impair Reaction Times?

- **Insight:**
  - **In practice, you should not delete outliers from a data set without sufficient cause (i.e., if it seems the observation was incorrectly recorded)**
  - **It is however, a good idea to check for sensitivity of an analysis to an outlier**
  - **If the results change much, it means that the inference including the outlier is on shaky ground**

Learning Objective 6:
Example: Females or males more nicotine dependent

- Test the claim that there is a difference between males and females and their level of dependence on nicotine with a level of significance of 1%

	Mean	S	N
Female	2.8	3.6	150
Male	1.6	2.9	182



We would reject the claim at a 1% level of significance

Learning Objective 6: Class exercise 1

- Many people take ginkgo supplements advertised to improve memory. Are these over the counter supplements effective?
- Based on the study results below, is there evidence that taking 40 mg of ginkgo 3 times a day is effective in increasing mean performance?
- Test the relevant hypothesis using α=5%

	n	Mean	S
Ginkgo	104	5.6	0.6
Placebo	115	5.5	0.6

Learning Objective 6: Class Exercise 2

- Attitude toward mathematics was measured for two different groups. The attitude scores range from 0 to 80 with the higher scores indicating a more positive attitude. One group consisted of Elementary education majors and the other group consisted of majors from several other areas. The results were as follows:

	N	mean	SD
Elementary Ed	75	42.7	15.5
Non Elem. Ed	110	49.3	17.0

- Calculate the P-value, and give your conclusion for testing H_0: $\mu_1 - \mu_2 = 0$, H_a: $\mu_1 - \mu_2 < 0$ at a level of significance equal to 0.05.