

Chapter 9: Comparing Two Groups

Section 9.3: Other Ways of Comparing Means and Comparing Proportions

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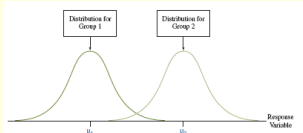
Learning Objectives

1. Alternative Method for Comparing Means: the Pooled Standard Deviation
2. Comparing Population Means, Assuming Equal Population Standard Deviations
3. Examples
4. The Ratio of Proportions: The Relative Risk

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Learning Objective 1: Alternative Method for Comparing Means

- An alternative t -method can be used when, under the null hypothesis, it is reasonable to expect the *variability* as well as the mean to be the same
- This method requires the assumption that the population standard deviations be equal



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Learning Objective 1: The Pooled Standard Deviation

- This alternative method estimates the common value σ of σ_1 and σ_2 by:

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

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Learning Objective 2:
Comparing Population Means, Assuming Equal
Population Standard Deviations

- Using the pooled standard deviation estimate, a 95% CI for $(\mu_1 - \mu_2)$ is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.025} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

- This method has $df = n_1 + n_2 - 2$

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Learning Objective 2:
Comparing Population Means, Assuming Equal
Population Standard Deviations

- The test statistic for $H_0: \mu_1 = \mu_2$ is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- This method has $df = n_1 + n_2 - 2$

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Learning Objective 2:
Comparing Population Means, Assuming Equal
Population Standard Deviations

- These methods assume:
 - Independent random samples from the two groups
 - An approximately normal population distribution for each group
 - This is mainly important for small sample sizes, and even then, the CI and the two-sided test are usually robust to violations of this assumption
- $\sigma_1 = \sigma_2$

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Learning Objective 3:

Example: Is Arthroscopic Surgery better than Placebo?

- Calculate the P-Value and determine if there is a statistical difference between Arthroscopic surgery and Placebo at 5% level of significance. Knee Pain Score

Group	Sample Size	Mean	Standard Deviation
1. Placebo	60	51.6	23.7
2. Arthroscopic—lavage	61	53.7	23.7

```

EDIT_CALC 1:5:16
1:2-Test...
2:1-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7:2.Interval...

2-SampTTest
Inpt:Data
n1:60
x1:51.6
Sx1:23.7
n2:61
Sx2:23.7
u1:u2
pooled:No
Calculate Draw

2-SampTTest
u1=u2
n1:60
x1:51.6
Sx1:23.7
n2:61
Sx2:23.7
u1:u2
pooled:No
Calculate Draw

2-SampTTest
u1=u2
n1:60
x1:51.6
Sx1:23.7
n2:61
Sx2:23.7
u1:u2
pooled:No
Calculate Draw
    
```

With a P-value of 0.63, we should not reject the null that there is no difference between placebo and Arthroscopic surgery

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Learning Objective 3:

Example: Is Arthroscopic Surgery better than Placebo?

- Calculate a 95% Confidence Interval

```

EDIT  CALC  TESTS
4:1-2-SampTInt...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
8:ZInterval...
9:2-SampTInt...
10:2-SampTInt...
11:2-SampTInt...

```

- We are 95% Confident that the difference between the placebo and surgery is in this range -10.6 to 6.4.
- Notice that 0 is within this range. Thus, we should not reject the null hypothesis at the 5% significance level that there is no difference between the two treatment groups

Learning Objective 3:

Example: Are Vegetarians More Liberal?

- Respondents were rated on a scale of 1-7 with 1 being liberal and 7 being the most conservative. Is there a significant difference between Non-vegetarian and vegetarians? Assume equal variances.

- $H_0: \mu_{(nveg)} = \mu_{(veg)}$ vs. $H_a: \mu_{(nveg)} \neq \mu_{(veg)}$

	Mean	S	N
Nonvegetarian	3.18	1.72	51
Vegetarian	2.22	0.67	9

```

2-SampTTest
Inpt:Data  [X]
x1:3.18
Sx1:1.72
n1:51
x2:2.22
Sx2:0.67
n2:9

```

Learning Objective 3:

Example: Are Vegetarians More Liberal?

Without assumption of equal variances:

```

2-SampTTest
μ1≠μ2
t=2.922735067
P=.0064516203
df=30.76965352
x1=3.18
Sx1=1.72

```

Depending on your assumption on whether the variance of both groups are equal or not impacts the conclusion of statistical significance.

Learning Objective 3:

Example: Are Vegetarians More Liberal?

- Calculate a 95% confidence interval

Assuming Equal Variances

```

2-SampTInt
Inpt:Data  [X]
x1:3.18
Sx1:1.72
n1:51
x2:2.22
Sx2:0.67
n2:9

```

```

2-SampTInt
Inpt:Data  [X]
x1:3.18
Sx1:1.72
n1:51
x2:2.22
Sx2:0.67
n2:9

```

```

2-SampTInt
(-.2097,2.1297)
df=58
x1=3.18
Sx1=1.72
x2=2.22
Sx2=0.67

```

```

2-SampTInt
(-.2097,2.1297)
x1=3.18
Sx1=1.72
x2=2.22
Sx2=0.67

```

Learning Objective 3:
Example: Are Vegetarians More Liberal?

- Assuming unequal variances, what is the 95% Confidence Interval?

```
2-SampTInt
n1:51
x1:2.22
s1:1.67
n2:9
C-Level: .95
Pooled: No
Calculate
```

```
2-SampTInt
(-.2899, 1.6301)
df=30.76965352
x1=3.18
x2=2.22
s1=1.72
s2=1.67
```

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Learning Objective 4:
The Ratio of Proportions: The Relative Risk

- The *ratio of proportions* for two groups is:

$$\frac{\hat{p}_1}{\hat{p}_2}$$

- In medical applications for which the proportion refers to a category that is an undesirable outcome, such as death or having a heart attack, this ratio is called the *relative risk*
- The ratio describes the sizes of the proportions *relative* to each other

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Learning Objective 4:
The Ratio of Proportions: The Relative Risk

- Recall Physician's Health Study:

TABLE 10.1: Whether Subject Had a Heart Attack, for Placebo and Aspirin Treatment Groups

Group	Heart Attack		Total
	Yes	No	
Placebo	189	10,845	11,034
Aspirin	104	10,933	11,037

$$\hat{p}_1 = 189/11034 = 0.0171$$

$$\hat{p}_2 = 104/11037 = 0.0094$$

$$\text{sample relative risk} = \hat{p}_1/\hat{p}_2 = 0.0171/0.0094 = 1.82$$

- The proportion of the placebo group who had a heart attack was 1.82 times the proportion of the aspirin group who had a heart attack.

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