Chapter 9: Comparing Two Groups

Section 9.4: How Can We Analyze Dependent Samples?

Learning Objectives

1. Dependent Samples
2. Example: Matched Pairs Design for Cell Phones and Driving Study
3. To Compare Means with Matched Pairs, Use Paired Differences
4. Confidence Interval For Dependent Samples
5. Paired Difference Inferences

Learning Objectives

6. Comparing Proportions with Dependent Samples
7. Confidence Interval Comparing Proportions with Matched-Pairs Data
8. McNemar's Test

Learning Objective 1: Dependent Samples

- **Each observation in one sample has a matched observation in the other sample**
- **The observations are called *matched pairs***

Learning Objective 2: Example: Matched Pairs Design for Cell Phones and Driving Study

- **The cell phone analysis presented earlier in this text used independent samples:**
  - **One group used cell phones**
  - **A separate control group did not use cell phones**

Learning Objective 2: Example: Matched Pairs Design for Cell Phones and Driving Study

- **An alternative design used the same subjects for both groups**
  - **Reaction times are measured when subjects performed the driving task without using cell phones and then again while using cell phones**

Learning Objective 2: Example: Matched Pairs Design for Cell Phones and Driving Study

Data:

The difference score is the reaction time using the cell phone minus the reaction time not using it, such as 636 − 604 = 32 milliseconds.

	Using Cell Phone?				Using Cell Phone?		
Student	No	Yes	Difference	Student	No	Yes	Difference
1	604	636	32	17	525	626	101
2	556	623	67	18	508	501	−7
3	540	615	75	19	529	574	45
4	522	672	150	20	470	468	−2
5	459	601	142	21	512	578	66
6	544	600	56	22	487	560	73
7	513	542	29	23	515	525	10
8	470	554	84	24	499	647	148
9	556	543	−13	25	448	456	8
10	531	520	−11	26	558	688	130
11	599	609	10	27	589	679	90
12	537	559	22	28	814	960	146
13	619	595	−24	29	519	558	39
14	536	565	29	30	462	482	20
15	554	573	19	31	521	527	6
16	467	554	87	32	543	536	−7

Learning Objective 2: Example: Matched Pairs Design for Cell Phones and Driving Study

- **Benefits of using dependent samples (matched pairs):**
  - **Many sources of potential bias are controlled so we can make a more accurate comparison**
  - **Using matched pairs keeps many other factors fixed that could affect the analysis**
  - **Often this results in the benefit of smaller standard errors**

Learning Objective 3: To Compare Means with Matched Pairs, Use Paired Differences

- **To Compare Means with Matched Pairs, Use Paired Differences:**
  - **For each matched pair, construct a difference score**
  - ***d* = (reaction time using cell phone) – (reaction time without cell phone)**
  - **Calculate the sample mean of these differences:** $\bar{x}_d$

Learning Objective 3: To Compare Means with Matched Pairs, Use Paired Differences

- **The difference ($\bar{x}_1 - \bar{x}_2$) between the means of the two samples equals the mean $\bar{x}_d$ of the difference scores for the matched pairs**
- **The difference ($\mu_1 - \mu_2$) between the population means is identical to the parameter μ_d that is the population mean of the difference scores**

Learning Objective 4: Confidence Interval For Dependent Samples

- **Let *n* denote the number of observations in each sample**
- **This equals the number of difference scores**
- **The 95 % CI for the population mean difference is:**

$$\bar{x}_d \pm t_{.025} \frac{s_d}{\sqrt{n}}$$

$\bar{x}_d$ is the sample mean of the differences

s_d is their standard deviation

Learning Objective 5: Paired Difference Inferences

- **These *paired-difference inferences* are special cases of single-sample inferences about a population mean so they make the same assumptions**

Learning Objective 5: Paired Difference Inferences

- **To test the hypothesis H_0: $\mu_1 = \mu_2$ of equal means, we can conduct the single-sample test of H_0: $\mu_d = 0$ with the difference scores**
- **The test statistic is:**

$$t = \frac{\bar{x}_d - 0}{\frac{s_d}{\sqrt{n}}} \text{ with } df = n - 1$$

Learning Objective 5: Paired Difference Inferences

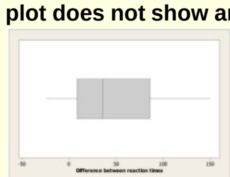
- **Assumptions:**
  - **The sample of difference scores is a random sample from a population of such difference scores**
  - **The difference scores have a population distribution that is approximately normal**
    - **This is mainly important for small samples (less than about 30) and for one-sided inferences**

Learning Objective 5: Paired Difference Inferences

- **Confidence intervals and two-sided tests are *robust*: They work quite well even if the normality assumption is violated**
- **One-sided tests do not work well when the sample size is small and the distribution of differences is highly skewed**

Learning Objective 5: Example: Cell Phones and Driving Study

- **The box plot shows skew to the right for the difference scores**
  - **Two-sided inference is robust to violations of the assumption of normality**
- **The box plot does not show any severe outliers**

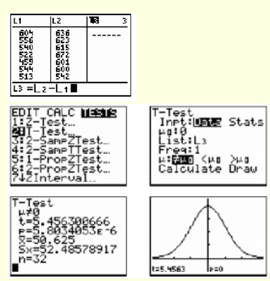


Learning Objective 5: Example: Cell Phones and Driving Study

- **Significance test:**
  - **H_0: $\mu_d = 0$ (and hence equal population means for the two conditions)**
  - **H_a: $\mu_d \neq 0$**
  - **Test statistic:**

$$t = \frac{50.6}{52.5/\sqrt{32}} = 5.46$$

Learning Objective 5: Example: Cell Phones and Driving Study



Learning Objective 5:
Example: Cell Phones and Driving Study

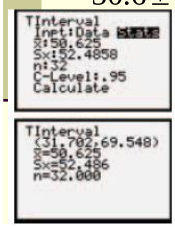
- **The P-value displayed in the output is approximately 0**
- **There is extremely strong evidence that the population mean reaction times are different**

Learning Objective 5:
Example: Cell Phones and Driving Study

- **95% CI for $\mu_d = (\mu_1 - \mu_2)$:**

$$50.6 \pm 2.040\left(\frac{52.5}{\sqrt{32}}\right) = 50.6 \pm 18.9$$

$$\text{or } (31.7, 69.5)$$



Learning Objective 5:
Example: Cell Phones and Driving Study

- **We infer that the population mean when using cell phones is between about 32 and 70 milliseconds higher than when not using cell phones**
- **The confidence interval is more informative than the significance test, since it predicts possible values for the difference**

Learning Objective 6:
Comparing Proportions with Dependent Samples

- A recent GSS asked subjects whether they believed in Heaven and whether they believed in Hell:

	Belief in Hell		
Belief in Heaven	Yes	No	**Total**
Yes	833	125	**958**
No	2	160	**162**
Total	**835**	**285**	**1120**

Learning Objective 6:
Comparing Proportions with Dependent Samples

- We can estimate p_1 - p_2 as:
  $\hat{p}_1 - \hat{p}_2 = 958/1120 - 835/1120 = 0.11$
- Note that the data consist of matched pairs.
  - Recode the data so that for belief in heaven or hell, 1=yes and 0=no

Heaven	Hell	Interpretation	Difference, *d*	Frequency
1	1	believe in Heaven and Hell	1-1=0	833
1	0	believe in Heaven, not Hell	1-0=1	125
0	1	believe in Hell, not Heaven	0-1=-1	2
0	0	do not believe in Heaven or Hell	0-0=0	160

Learning Objective 6:
Comparing Proportions with Dependent Samples

- Sample mean of the 1120 difference scores is

  [0(833)+1(125)-1(2)+0(160)]/1120=0.11
- Note that this equals the difference in proportions $\hat{p}_1 - \hat{p}_2$
- We have converted the two samples of binary observations into a single sample of 1120 difference scores. We can now use single-sample methods with the differences as we did for the matched-pairs analysis of means.

Learning Objective 7:
Confidence Interval Comparing Proportions with Matched-Pairs Data

- Use the fact that the sample difference $\hat{p}_1 - \hat{p}_2$ is the mean of difference scores of the re-coded data
- We can then find a confidence interval for the population mean of difference scores using single sample methods

Learning Objective 7:
Confidence Interval Comparing Proportions with Matched-Pairs Data

- $n = 1120$

  $\bar{x}_d = 0.1098$

  $s_d = 0.3185$

  $$95\%\ \text{CI} = 0.1098 \pm 1.96\left(0.3185/\sqrt{1120}\right)$$
  $$= 0.1098 \pm 0.0187$$
  $$= (0.091, 0.128)$$

Learning Objective 8:
McNemar Test for Comparing Proportions with Matched-Pairs Data

- Hypotheses: H_0: $p_1 = p_2$, H_a can be one or two sided
- Test Statistic: For the two counts for the frequency of "yes" on one response and "no" on the other, the z test statistic equals their difference divided by the square root of their sum.
- P-value: The probability of observing a sample even more extreme than the observed sample

Learning Objective 8:
McNemar Test for Comparing Proportions with Matched-Pairs Data

- Assumptions:
  - The sum of the counts used in the test should be at least 30, but in practice, the two-sided test works well even if this is not true.

Learning Objective 8:
Example: McNemar's Test

- Recall GSS example about belief in Heaven and Hell:

	Belief in Hell		
Belief in Heaven	Yes	No	**Total**
Yes	833	125	**958**
No	2	160	**162**
Total	**835**	**285**	**1120**

Learning Objective 8:
Example: McNemar's Test

- McNemar's Test:

  $$z = \frac{125 - 2}{\sqrt{125 + 2}} = 10.9$$

  P-value is approximately 0.

- Note that this result agrees with the confidence interval for p_1-p_2 calculated earlier