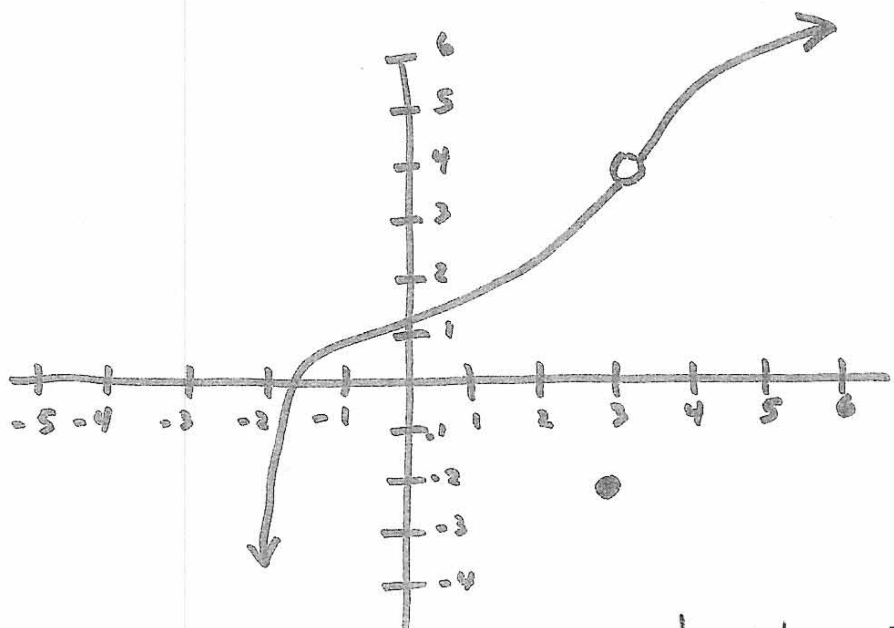
Chapter 1: Section 1.1 and 1.2

Definition: If x approaches c $(x \neq c)$, the <u>limit of f(x)</u> is the y-coordinate L that f(x) tends to. This is denoted $\lim_{x \to c} f(x) = L$. In addition, the limit must be unique.

Ex. 1



$$\lim_{x \to 3} f(x) = 4$$

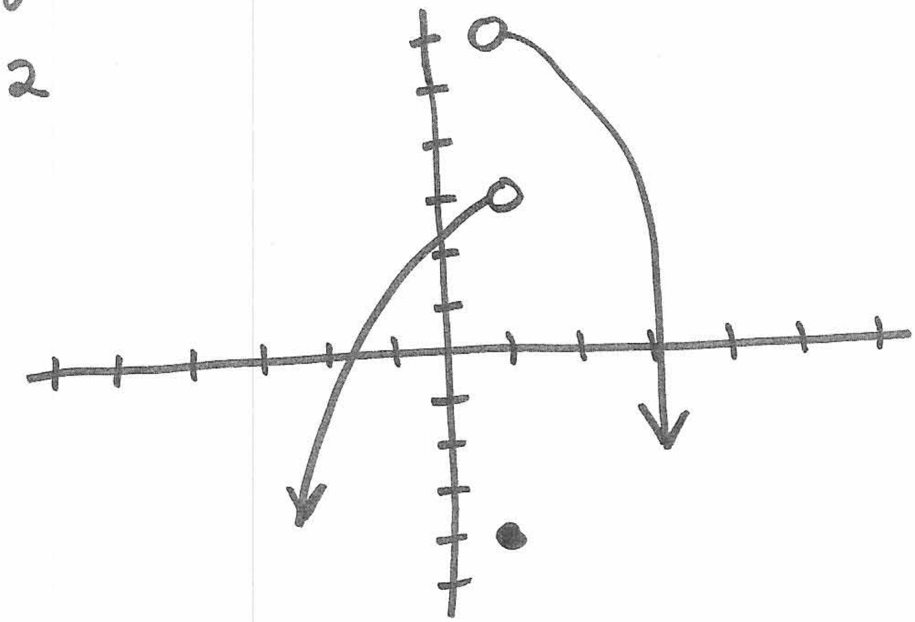
$$f(3) = -2$$

Similar Example

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

Uniqueness

Ex. 2



$\lim_{x \to 1^-} f(x) = 3$ (Approaching 1 from the left : .999)

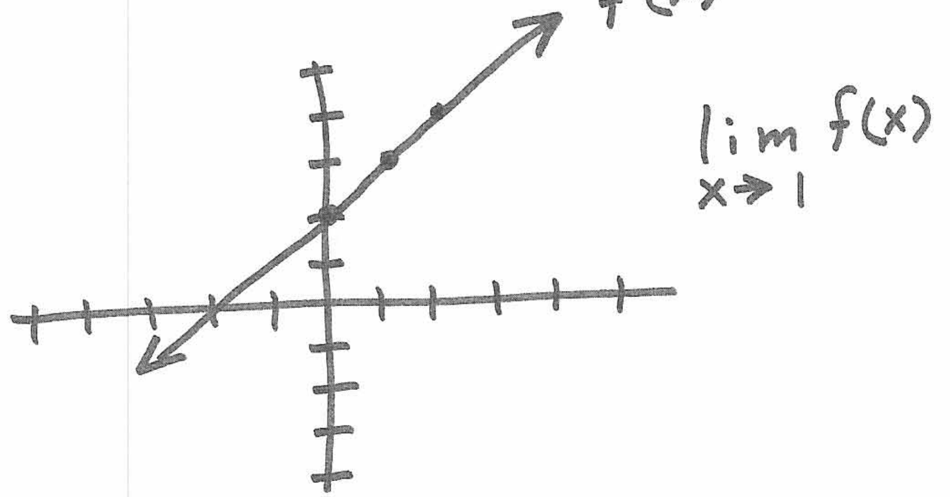
$\lim_{x \to 1^+} f(x) = 6$ (Approaching 1 from the right : 1.0001)

$f(1) = -4$

$\lim_{x \to 1} f(x)$ does not exist

Observation: If $f(x)$ is continuous on an open interval containing c then $\lim_{x \to c} f(x) = f(c)$.

Ex. 3



$$\lim_{x \to 1} f(x) = 3$$

Ex. 4 $\lim_{x \to 4} (x^2 + 6x - 7) = 16 + 24 - 7 = 33$

Ex. 5 $\lim_{x \to 3} \frac{2x+1}{x-3}$ does not exist

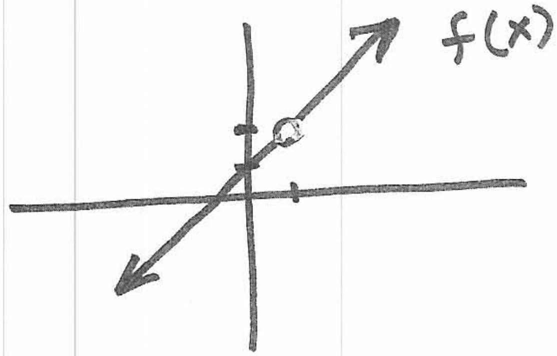
$$\lim_{x \to 3^-} \frac{2x+1}{x-3} = -\infty$$

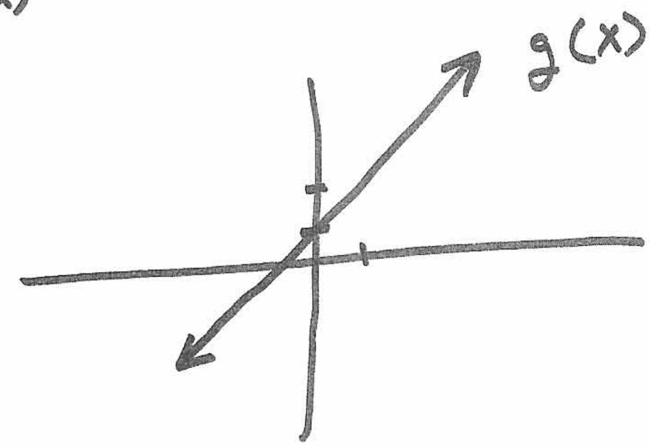
$$\lim_{x \to 3^+} \frac{2x+1}{x-3} = \infty$$

Observation: If $f(x) = g(x)$ on an open interval containing c and $f(c) \neq g(c)$ then $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} g(x)$.

Ex. 6 $\quad \lim\limits_{x \to 1} \left[\overbrace{\frac{x^2 - 1}{x - 1}}^{f(x)} = \frac{(x+1)(x-1)}{x-1} \right] =$

$\lim\limits_{x \to 1} \underbrace{(x+1)}_{g(x)} = 2$





Ex. 7 $\quad \lim\limits_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4} =$

$\lim\limits_{x \to 2} \left[\frac{(x+1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} \right] = \frac{3}{4}$

Ex. 8 $\lim_{x \to \infty} \frac{1}{x} = 0$

Recall, $x^{-1} = \frac{1}{x}$

Observation: $\lim_{x \to \infty} x^{-n} = 0$ if $n > 0$

Ex. 9 $\lim_{x \to \infty} \frac{10x^2 - x + 3}{5x^2 - 8x + 11} =$

$$\lim_{x \to \infty} \frac{10x^2 - x + 3}{5x^2 - 8x + 11} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$$

$$\lim_{x \to \infty} \frac{10 - \frac{1}{x} + \frac{3}{x^2}}{5 - \frac{8}{x} + \frac{11}{x^2}} = \frac{10 - 0 + 0}{5 - 0 + 0}$$

$$= 2$$

Ex. 10 $\lim_{x \to \infty} \frac{x^3 - 5}{x^2 + 2x} =$

$$\lim_{x \to \infty} \left[\frac{x^3 - 5}{x^2 + 2x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{x - \frac{5}{x^2}}{1 + \frac{2}{x}} \right] =$$

$$\frac{\infty - 0}{1 + 0} = \infty$$

Ex. 11 $\lim_{x\to\infty} R\left[\frac{1-(1.04)^{-x}}{.04}\right] =$

$R\lim_{x\to\infty}\left[\frac{1}{.04}-\frac{1}{.04(1.04)^{x}}\right] =$

$R\lim_{x\to\infty}\left(\frac{1}{.04}\right) = 25R.$

Ex. 12 $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} =$

$\lim_{x\to 9}\frac{x^{\frac{1}{2}}-3}{x-9} =$

$\lim_{x\to 9}\frac{\cancel{(x^{\frac{1}{2}}-3)}}{(x^{1/2}+3)\cancel{(x^{1/2}-3)}} =$

$\lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$

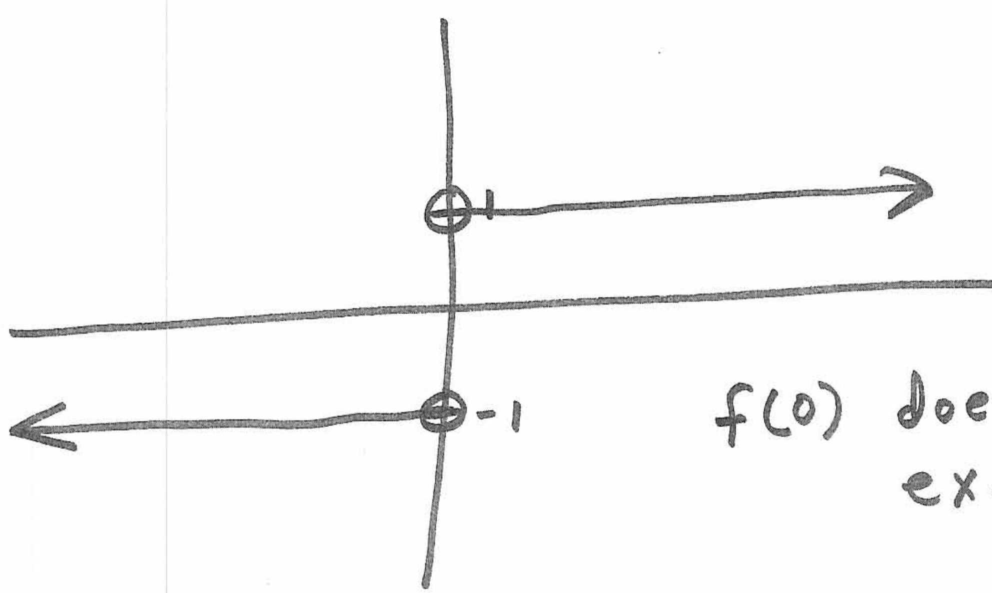
Ex. 13 $\quad \lim_{x\to 0} \frac{|x|}{x}$

Recall, $\quad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \left[\frac{x}{x} = 1\right] = 1$$

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \left[\frac{-x}{x} = -1\right] = -1$$

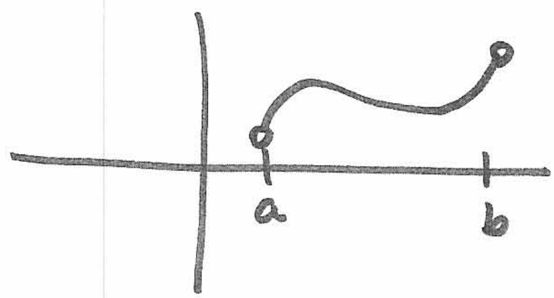
$\lim_{x\to 0} \frac{|x|}{x}$ does not exist



f(0) does not exist

Continuity

In layman's terms, a function $f(x)$ is continuous on the open interval (a, b) if you can draw the function without picking up your writing utensil from a to b.



Formally, $f(x)$ is continuous on (a,b) if for every c in (a,b) <u>all</u> of the following are true

1) $f(c)$ is defined
2) $\lim\limits_{x \to c} f(x)$ exists
3) $\lim\limits_{x \to c} f(x) = f(c)$

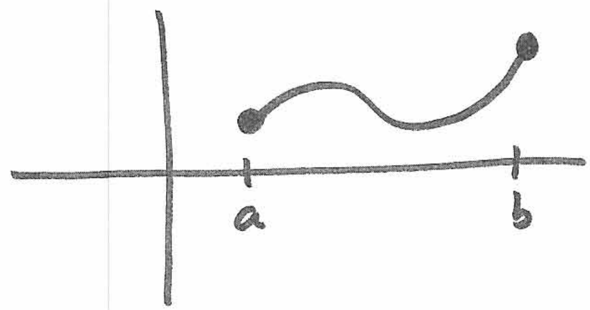
If one of these is not true then c is called a point of discontinuity.

If in addition,

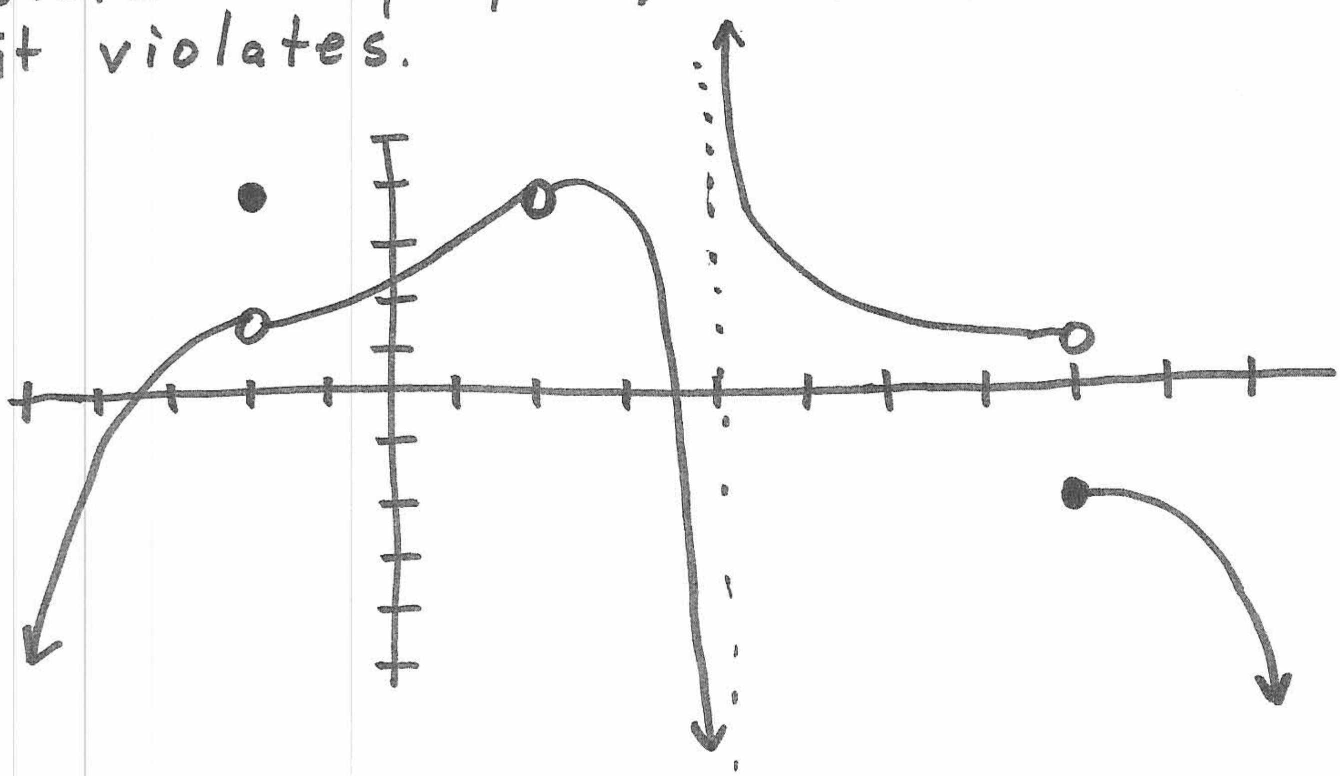
4) $\lim_{x \to a^+} f(x) = f(a)$

5) $\lim_{x \to b^-} f(x) = f(b)$

then $f(x)$ is continuous on $[a, b]$

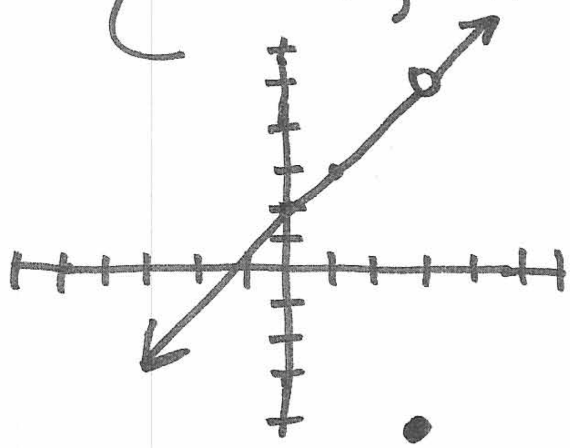


Find the point(s) of discontinuity. State the property or properties it violates.



If a function can be made continuous on (a, b) by redefining one point c then c is a point of removable discontinuity.

$$f(x) = \begin{cases} x+2, & x \neq 3 \\ -4, & x = 3 \end{cases}$$



$x = 3$ is a point of removable discontinuity.

Sketch

$$f(x) = \begin{cases} 3-x, & x < -2 \\ 1-x^2, & x \geq -2 \end{cases}$$

Find k so that $f(x)$ is continuous

$$f(x) = \begin{cases} kx^2, & x \le 2 \\ x + k, & x > 2 \end{cases}$$