1.3 Average rate of change $= \dfrac{f(b) - f(a)}{b - a}$

A car finishes a race. Its distance in feet from the finish line after crossing it t second earlier is $s(t) = 160t - 5t^2$ where $0 \le t \le 16$.

Find the average velocity = average rate in change of distance from t_1 to t_2 if t_1 and t_2 are

a) 8, 16 : $\text{Avg} = \dfrac{s(16) - s(8)}{16 - 8} = \dfrac{1280 - 960}{8} = 40\text{ ft/sec}$

b) 8, 12 : $\text{Avg} = \dfrac{s(12) - s(8)}{12 - 8} = \dfrac{1200 - 960}{4} = 60\text{ ft/sec}$

c) 8, 9 : $\text{Avg} = \dfrac{s(9) - s(8)}{9 - 8} = \dfrac{1035 - 960}{1} = 75\text{ ft/sec}$

d) 8, 8.5 : $\text{Avg} = \dfrac{s(8.5) - s(8)}{8.5 - 8} = \dfrac{998.75 - 960}{0.5} = 77.5\text{ ft/s}$

e) 8, 8.1 : $\text{Avg} = \dfrac{s(8.1) - s(8)}{8.1 - 8} = \dfrac{967.95 - 960}{0.1} = 79.5\text{ ft/se}$

f) 8, 8.001 : $\text{Avg} = \dfrac{s(8.001) - s(8)}{8.001 - 8} = \dfrac{960.079995 - 960}{0.001}$

$= 79.995\text{ ft/sec}$

Instantaneous Rate of Change =

$$\lim_{(a-b)\to 0} \frac{f(b)-f(a)}{b-a} \; ; \text{ let } h = b-a \implies b = a+h$$

Usual formula is

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

What will the speedometer display when $t = 8$ seconds?

Instantaneous Speed =

$$\lim_{h\to 0} \frac{s(8+h)-s(8)}{h} \; ; \quad s(t) = 160t - 5t^2$$

$$s(8) = 160(8) - 5(8)^2 = 960$$

$$\begin{aligned} s(8+h) &= 160(8+h) - 5(8+h)^2 \\ &= 1280 + 160h - 5(64+16h+h^2) \\ &= 1280 + 160h - 320 - 80h - 5h^2 \\ &= 960 + 80h - 5h^2 \end{aligned}$$

$$\lim_{h\to 0} \frac{s(8+h) - s(8)}{h} =$$

$$\lim_{h\to 0} \frac{(960 + 80h - 5h^2) - 960}{h} =$$

$$\lim_{h\to 0} \left[\frac{80h - 5h^2}{h} = \frac{h(80-5h)}{h} = 80 - 5h \right] =$$

$$\frac{80 \text{ ft}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{5280 \text{ ft}} = 54.\overline{54} \text{ mph}$$

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Find an equation for the ~~velocity~~ speed; i.e., ~~that~~ any t, not just $t = 8$ seconds.

$$\text{Speed} = \lim_{h\to 0} \frac{s(t+h) - s(t)}{h}$$

Recall, $s(t) = 160t - 5t^2$

$$s(t+h) = 160(t+h) - 5(t+h)^2$$

$$= 160t + 160h - 5(t^2 + 2th + h^2)$$

$$= 160t + 160h - 5t^2 - 10th - 5h^2$$

Speed = v =

$$\lim_{h \to 0} \frac{(160t + 160h - 5t^2 - 10th - 5h^2) - (160t - 5t^2)}{h}$$

$$\lim_{h \to 0} \left[\frac{160h - 10th - 5h^2}{h} = \frac{h(160 - 10t - 5h)}{h} \right] =$$

$$\lim_{h \to 0} 160 - 10t - 5h = \underline{\underline{160 - 10t.}}$$

Thus, $v(8) = 160 - 10(8) = 80$ ft/sec

What is the velocity when $t = 2$? $t = 6$?

$v(2) = 140$ ft/sec , $v(6) = 100$ ft/sec

What is the velocity when the car crosses the finish line?

$$v(0) = 160 \text{ ft/sec} * \frac{3600 \text{ sec}}{5280 \text{ ft}} = 109.\overline{09} \text{ mph}$$

How far is the car from the finish line when it is at a complete stop?

Sol

Set $v(t) = 0$. $\quad 160 - 10t = 0 \Rightarrow t = 16$

$$s(16) = 160(16) - 5(16)^2 = 1280 \text{ feet.}$$

28. ***Revenue*** The revenue (in thousands of dollars) from producing x units of an item is

$$R(x) = 10x - .002x^2.$$

a. Find the average rate of change of revenue when production is increased from 1000 to 1001 units.

b. Find the instantaneous rate of change of revenue with respect to the number of items produced when 1000 units are produced. (This number is called the *marginal revenue* at $x = 1000$.)

c. Find the additional revenue if production is increased from 1000 to 1001 units.

d. Compare your answers for parts a and b. What do you find?