

## 1.5 Shortcuts

If  $y = f(x)$ , the derivative can be denoted  $f'(x)$ ,  $y'$ ,  $\frac{dy}{dx}$ ,  $\frac{d}{dx} f(x)$ ,  $D_x(y)$ ,  $D_x[f(x)]$

### Constant Rule

If  $f(x) = k$  then  $f'(x) = 0$

Of course, the slope of a horizontal line is ZERO.

Ex. If  $y = 4$  then  $\frac{dy}{dx} = 0$

### Power Rule

If  $f(x) = x^n$  then  $f'(x) = nx^{n-1}$

Ex. If  $f(x) = x^9$  then  $\frac{d}{dx}(x^9) = 9x^8$

## Scalar Rule

$$D_x [k f(x)] = k f'(x)$$

Ex. If  $f(x) = 8x^5$   
then  $f'(x) = 8 \cdot 5x^4 = 40x^4$

Ex. If  $f(x) = 6x^3 - 3x^2 + x - 13$   
then  $f'(x) = 18x^2 - 6x + 1$

Ex.  $f(x) = \frac{1}{x}$

Ex.  $f(x) = \sqrt{x} - \sqrt[3]{x}$

Ex.  $p(t) = \frac{3 + t^3 - \sqrt{t}}{t}$

Ex.  $y = \frac{50}{\sqrt{x}} - 7x^{10}$

Ex.  $g(w) = (3w - 5)^2$

## Demand Function, $P(x)$

$P(x)$  is the price that causes  $x$  items to be purchased.

Ex. If  $P(x) = \frac{200 - x}{100}$   
then 50 items will be purchased  
if the price is \$1.50.

## Revenue Function, $R(x)$

$R(x) = xP(x)$ . It is the amount of money received from the sale of  $x$  items.

Ex. If  $P(x) = \frac{200 - x}{100}$   
then  $R(x) = xP(x) = \frac{200x - x^2}{100}$  and  
 $R(50) = 50(\$1.50) = \$75$ .

## Cost Function, $C(x)$

$C(x)$  is the cost to make  $x$  items

Ex. If  $C(x) = .30x + 20$   
then  $C(50) = \$35$

## Profit Function, $P(x)$

$P(x)$  is the profit made from selling  $x$  items

$$P(x) = R(x) - C(x)$$

Ex. Using previous examples,  $P(50) = 75 - 35 = \$40$

Marginal Profit  
Cost Revenue is "almost" the additional  
money derived from increasing production by  
one unit.

$$\text{Marginal Profit} = P'(x)$$

$$\text{Marginal Cost} = C'(x)$$

$$\text{Marginal Revenue} = R'(x)$$

Ex. Let  $R(x) = 2x - \frac{1}{25,000}x^2$

a) Find  $R(10,001) - R(10,000)$

b) Find  $R'(10,000)$

c) Find  $R'(35,000)$

d) Find  $x$  so that  $R(x)$  is maximum.

Ex. Say the demand function is  $p(x) = 3 - \frac{x}{15,000}$  and the cost function is a linear function where the fix cost is \$300 and the cost of making one item is \$0.75.

a) Find  $R(x)$

b) Find  $C(x)$

c) Find  $P(x)$

d) Find  $x$  so that  $P(x)$  is maximum.

e) What price should they sell the item for?