1.6 Product Rule and Quotient Rule

Product Rule: If $f(x) = u \cdot v$
then $f'(x) = uv' + vu'$

Example: Let $f(x) = \underbrace{(3x^7 + 8)}_{u(x)}\underbrace{(5x^4 - 6)}_{v(x)}$.

$$f'(x) = (3x^7+8)(20x^3) + (5x^4-6)(21x^6)$$
$$= 60x^{10} + 160x^3 + 105x^{10} - 126x^6$$
$$= 165x^{10} - 126x^6 + 160x^3$$

Alt. $f(x) = (3x^7+8)(5x^4-6)$
$$= 15x^{11} - 18x^7 + 40x^4 - 48 \Rightarrow$$
$$f'(x) = 165x^{10} - 156x^6 + 160x^3$$

euler constant (pronounced "oiler")

Strange Number, $e \approx 2.71828182846$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = x^3 e^x$ find $f'(x)$

$$f'(x) = x^3 e^x + e^x(3x^2)$$
$$= x^2 e^x (x+3)$$

If $f(x) = e^{2x}$ find $f'(x)$

$$f(x) = e^x e^x \Rightarrow f'(x) = e^x e^x + e^x e^x$$
$$= e^{2x} + e^{2x} = 2e^{2x}$$

Quotient Rule: If $f(x) = \dfrac{u}{v}$

then $f'(x) = \dfrac{vu' - uv'}{v^2}$

Example: Let $f(x) = \dfrac{3x+8}{4x-1}$

$$f'(x) = \frac{(4x-1)(3) - (3x+8)(4)}{(4x-1)^2}$$

$$= \frac{12x - 3 - (12x+32)}{(4x-1)^2} = \frac{12x-3-12x-32}{(4x-1)^2}$$

$$= \frac{-35}{(4x-1)^2}$$

$$y = \frac{\sqrt[5]{x^2}}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1)\cdot \frac{2}{5}x^{-3/5} - x^{2/5}}{(x+1)^2} \cdot \frac{5}{5}$$

$$= \frac{2(x+1)x^{-3/5} - 5x^{2/5}}{5(x+1)^2}$$

x is common to both terms on top. Take out the one with smaller exponent

$$= \frac{x^{-3/5}\left[2(x+1) - 5x\right]}{5(x+1)^2}$$

note:

$$\frac{5x^{2/5}}{x^{-3/5}} = 5x^{\frac{2}{5}-(-3/5)} = 5x^{5/5} = 5x$$

$$= \frac{x^{-3/5}\left[2x+2-5x\right]}{5(x+1)^2}$$

$$= \frac{2-3x}{5x^{3/5}(x+1)^2}$$

Let $f(x) = \dfrac{\sqrt{x}}{x-8}$

$$f'(x) = \frac{(x-8)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(1)}{(x-8)^2} \cdot \frac{2}{2}$$

$$= \frac{x^{-\frac{1}{2}}(x-8) - 2x^{1/2}}{2(x-8)^2}$$

$$= \frac{x^{-\frac{1}{2}}\left[(x-8) - 2x\right]}{2(x-8)^2}$$

$$= \frac{x^{-\frac{1}{2}}(-x-8)}{2(x-8)^2} = \frac{-x^{-\frac{1}{2}}(x+8)}{2(x-8)^2}$$

$$= \frac{-(x+8)}{2\sqrt{x}\,(x-8)^2}$$

Let $f(x) = \dfrac{e^x}{x}$

$$f'(x) = \frac{xe^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$$

Let $f(x) = \dfrac{xe^x}{x-1}$

$$f'(x) = \frac{(x-1)\left[xe^x + e^x\right] - xe^x(1)}{(x-1)^2}$$

$$= \frac{(x-1)e^x(x+1) - xe^x}{(x-1)^2}$$

$$= \frac{e^x\left[(x-1)(x+1) - x\right]}{(x-1)^2}$$

$$= \frac{e^x(x^2-1-x)}{(x-1)^2}$$