1.7 Chain Rule : $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Let $y = e^{5x^3}$, Find $\frac{dy}{dx}$

let $u = 5x^3 \Rightarrow y(x) = (y \circ u)(x)$

$y = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 15x^2 = 15x^2 e^{5x^3}$$

Let $y = (7x^2 + 5x - 9)^2$, Find $\frac{dy}{dx}$

Without Chain Rule :

$$\begin{array}{r} 7x^2 + 5x - 9 \\ 7x^2 + 5x - 9 \\ \hline 49x^4 + 35x^3 - 63x^2 \qquad\qquad\quad \\ +35x^3 + 25x^2 - 45x \qquad \\ -63x^2 - 45x + 81 \\ \hline y = 49x^4 + 70x^3 - 101x^2 - 90x + 81 \end{array}$$

$$\frac{dy}{dx} = 196x^3 + 210x^2 - 202x - 90$$

let $y = u^2$, $u = 7x^2 + 5x - 9$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u(14x + 5)$$

$$= 2(7x^2 + 5x - 9)(14x + 5)$$

Generalized Power Rule

If $y = [g(x)]^n$

let $u = g(x)$

$y = u^n$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = nu^{n-1} g'(x) = n[g(x)]^{n-1} g'(x)$$

- - - -

Let $y = (7x^2 + 5x - 9)^2$, find $\frac{dy}{dx}$ using G.P.R.

$$\frac{dy}{dx} = 2(7x^2 + 5x - 9)(14x + 5)$$

or

$$\begin{array}{r} 14x^2 + 10x - 18 \\ 14x + 5 \\ \hline 196x^3 + 140x^2 - 252x \\ 70x^2 + 50x - 90 \\ \hline 196x^3 + 210x^2 - 202x - 90 \end{array}$$

Let $f(x) = (2x^2 - 9x + 18)^{100}$, find $f'(x)$

$$f'(x) = 100(2x^2 - 9x + 18)^{99}(4x - 9)$$

Let $f(x) = \sqrt{5x^3 - x^2} = (5x^3 - x^2)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(5x^3 - x^2)^{-\frac{1}{2}}(15x^2 - 2x)$$

$$= \frac{15x^2 - 2x}{2\sqrt{5x^3 - x^2}}$$

Let $f(x) = x\sqrt[3]{x+1} = x(x+1)^{1/3}$

$$f'(x) = x\left[\frac{1}{3}(x+1)^{-2/3}\right] + (x+1)^{1/3}(1)$$

$$= (x+1)^{-2/3}\left[\frac{1}{3}x + (x+1)\right]$$

$$= \frac{\frac{4}{3}x + 1}{(x+1)^{2/3}} \cdot \frac{3}{3} = \frac{4x+3}{3(x+1)^{2/3}}$$

Let $f(x) = \dfrac{5}{(2x-9)^6}$

Let $f(x) = \dfrac{1}{\sqrt{x^2+5}}$

1.8 Higher-Order Derivatives

	Newton	Leibnitz
1st	$y', f'(x)$	dy/dx
2nd	$y'', f''(x)$	d^2y/dx^2
3rd	$y''', f'''(x)$	d^3y/dx^3
4th	$y^{(4)}, f^{(4)}(x)$	d^4y/dx^4
5th	$y^{(5)}, f^{(5)}(x)$	d^5y/dx^5

Kinematics - Study of Motion

Change in rate of displacement ($\approx$ distance) is velocity ($\approx$ speed)

If $s(t)$ is the displacement then $v(t) = s'(t) = ds/dt$ is velocity. Speed $= |v|$.

the change in rate of velocity.
$$\left\{\begin{aligned} a(t) &= v'(t) = dv/dt \\ &= s''(t) = d^2s/dt^2 \end{aligned}\right.$$

Factoid: $a'(t) = v''(t) = s'''(t)$ is called jerk.

Ex. Find $f'(x)$, $f''(x)$, $f'''(x)$, & $f^{(4)}(x)$ if
$f(x) = 5x^3 - x^2 - \frac{1}{x}$.

Ex. Free-Fall Equation:
$s(t) = -\frac{1}{2}at^2 + v_0 t + s_0$

If a rocket shot from a 300-ft building with an initial velocity of 240 ft/sec then find

a) $s(t)$

b) the height when $t = 5$ seconds.

c) how long it is airbourne.

d) velocity when $t = 5$ sec & $t = 12$ sec.

e) max height above the ground.

f) acceleration at any time

g) jerk at any time