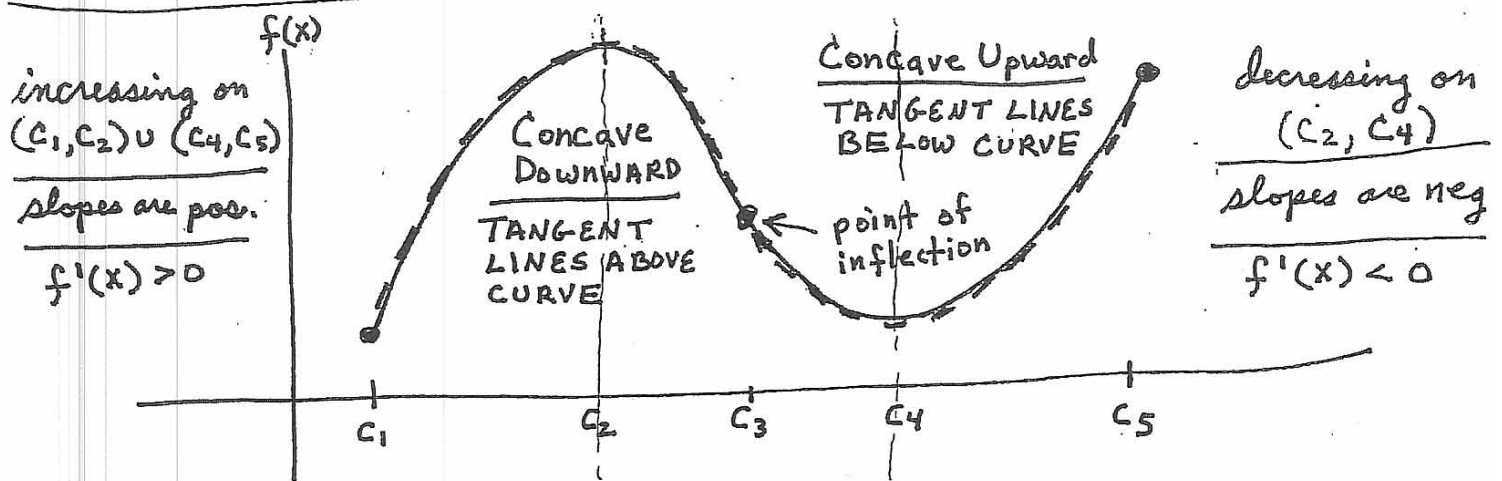


2.2 & 2.3

# CONCAVITY



CONCAVE DOWNWARD ON  
 $(c_1, c_3)$ ; slopes of  
 tangent lines are decreasing  
 $f''(x) < 0$

CONCAVE UPWARD ON  
 $(c_3, c_5)$ ; slopes of  
 tangent lines are increasing  
 $f''(x) > 0$

Note:  $f(c_2)$  is a relative max and  
 $f''(c_2) < 0$

$f(c_4)$  is a relative min and  
 $f''(c_4) > 0$

2<sup>nd</sup> Derivative Test says if  
 $c$  is a critical value and

1)  $f''(c) < 0$ ,  $f(c)$  is a rel. max

2)  $f''(c) > 0$ ,  $f(c)$  is a rel. min

3)  $f''(c) = 0$ , we ~~do~~ use 1<sup>st</sup> derivative test.

Ex. Let  $f(x) = \frac{6}{x^2+3} = 6(x^2+3)^{-1}$ . Determine the open intervals where  $f(x)$  is concave upward or downward

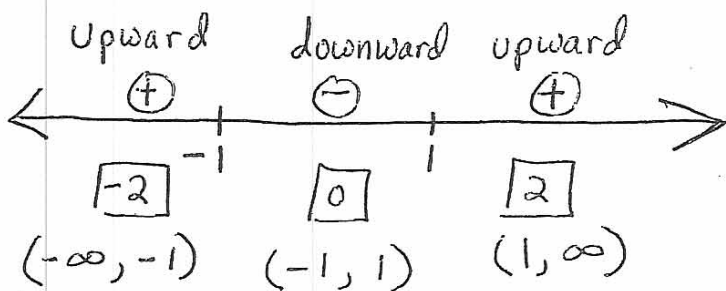
$$f'(x) = -6(x^2+3)^{-2}(2x) = \frac{-12x}{(x^2+3)^2}$$

$$f''(x) = \frac{(x^2+3)^2(-12) + 12x[2(x^2+3)(2x)]}{(x^2+3)^4}$$

$$= \frac{12(x^2+3)[-(x^2+3) + 4x^2]}{(x^2+3)^4} = \frac{12(3x^2-3)}{(x^2+3)^3}$$

$$= \frac{36(x^2-1)}{(x^2+3)^3} = \frac{36(x+1)(x-1)}{(x^2+3)^3}$$

CN<sup>''</sup>: -1, 1



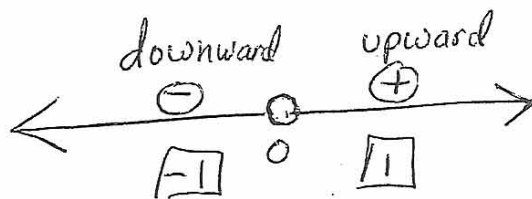
Since the concavity changed and  $f(-1)$  and  $f(1)$  exist, we have two inflection points  $(-1, 1.5)$  &  $(1, 1.5)$

Ex.  $f(x) = \frac{1}{x} = x^{-1}$  [Note:  $f(0)$  dne]

$$f'(x) = \frac{-1}{x^2} = -x^{-2}$$

$$f''(x) = \frac{2}{x^3}$$

CN<sup>''</sup>: none



no inflection point because  $f(0)$  does not exist.

Ex. Let  $f(x) = x^4 - 4x^3 = x^3(x-4)$

1) Find the  $x$  and  $y$  intercepts

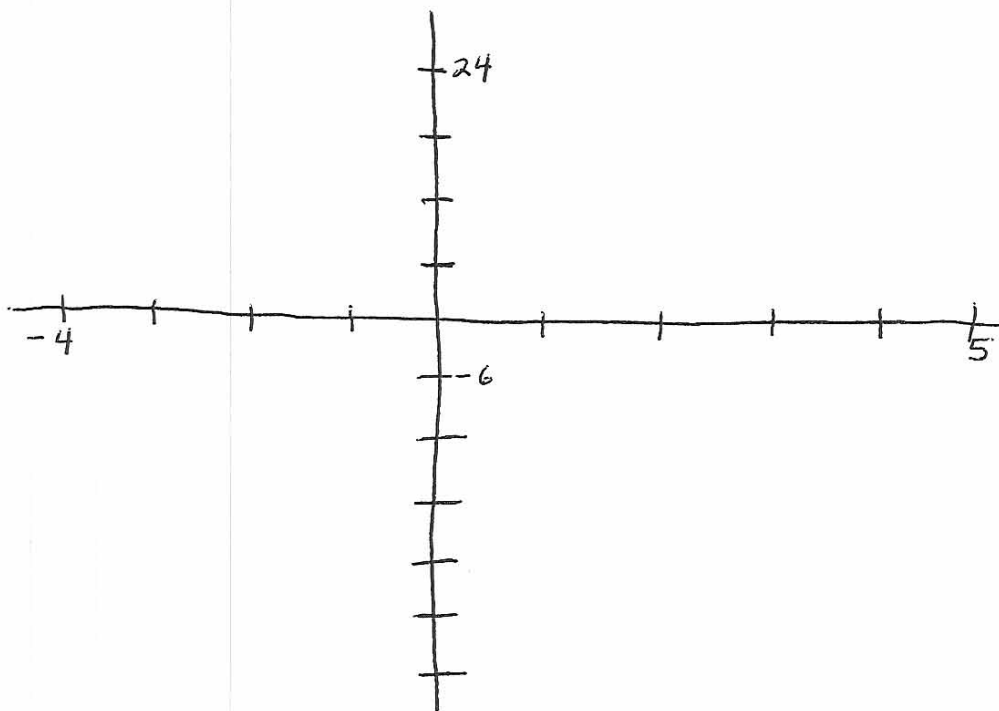
2) Where is  $f(x)$  increasing/decreasing

3) find the rel. extrema

4) Where is  $f(x)$  concave upward/downward

5) find inflection points

6) Sketch



In some applications, the inflection point is the point of diminishing return.

Ex. A company has determined that an investment of  $x$  hundred dollars in advertisement results in higher revenue;  $R(x) = 4.5\sqrt[3]{x-10} + 10$  is the daily revenue in thousands of dollars. Notice from its graph, it is an increasing function; also, beyond the inflection point, the company gets less and less "bang" for the buck. Find the point of diminishing return. The company will probably use this to determine how much they will spend in advertising.

$$\begin{array}{l|l} x_{\min} = 0 & y_{\min} = 0 \\ x_{\max} = 20 & y_{\max} = 20 \end{array}$$

Consider  $f(x) = \frac{3x+1}{x-1}$ . Sketch

after performing the following steps.

- 1) find x and y-intercepts
- 2) find the asymptotes
- 3) where is the function increasing/decreasing
- 4) find any relative extrema
- 5) where is the function concave upward/downward
- 6) find any pts of inflections.

Sketch  $\frac{x^2 + 1}{x}$  without the aid of your  
calculator