3.6 Elasticity of Demand

How will a small change in price affect the demand? Δ, greek letter delta, mean difference or change in

That is, $$\frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q/q}{\Delta p/p}$$

$$= \frac{\Delta q}{q} \cdot \frac{p}{\Delta p}$$

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As $\Delta p \to 0$, this becomes

$$\frac{p}{q} \cdot \frac{dq}{dp}.$$

Elasticity of demand is

$$\mathcal{E} = -\frac{p}{q} \cdot \frac{dq}{dp}$$ (notice, the equation has a negative sign)

Why? - As price goes up, demand goes down.

Example: Demand (daily) for DVD rentals is given by $q(p) = 120 - 20p$.

a) Find $\mathcal{E}(p)$

$$\mathcal{E} = \frac{-p}{q} \cdot \frac{dq}{dp} = \frac{-p}{120-20p}(-20)$$

$$= \frac{20p}{20(6-p)} = \frac{p}{6-p}$$

b) What is the quantity demanded when the price is \$2.00?

$$q(2.00) = 120 - 20(2.00) = 80$$

c) Find $\mathcal{E}$ at \$2.00

$$\mathcal{E}(2) = \frac{2}{6-2} = \frac{1}{2}.$$

this means if the price when up 1% when $p = \$2.00$ the demand would go down approximately $\frac{1}{2}\% = .005$.

That is, if the price rose 1% of $2 = 2¢ then quantity would decrease to approximately

$80(.995) = 79.6$

d) Find quantity demanded at $2.02.

$$q(2.02) = 120 - 20(2.02) = 79.6$$

e) Find ξ at $4.00

f) At what price is $\xi = 1$?

If $\xi < 1$ then revenue is increasing as the price increases [demand is Inelastic]

If $\xi > 1$ then revenue is decreasing as the price increases [demand is Elastic]

We have <u>unit elasticity</u> when $\xi = 1$.
Revenue is maximum when $\xi = 1$.

Example: The world's demand for oil can be described by $D(x) = 63{,}000 + 50x - 25x^2$, $0 \le x \le 50$, where D is in millions of barrels of oil per day at price $x per barrel.

a) Find the elasticity of demand function.

b) Find $\mathcal{E}$ at \$10 per barrel.

c) Elastic or Inelastic

d) Find $\mathcal{E}$ at \$20 ; Elastic or Inelastic

e) Find $\mathcal{E}$ at \$30 ; Elastic or Inelastic

f) At what price is revenue max?

g) What quantity of oil will be sold when revenue is max?

h) At \$30 per barrel, will a small increase in price result in an increase or a decrease in revenue?