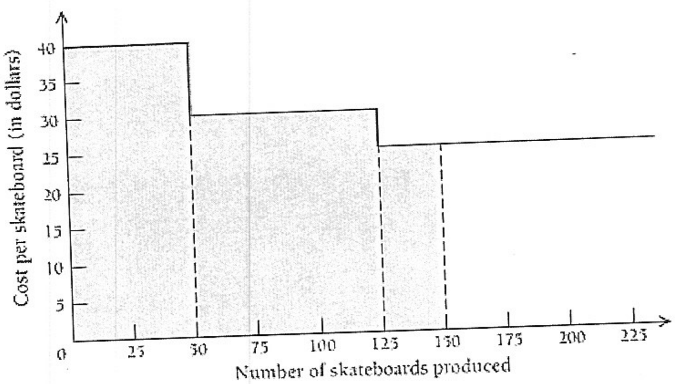
$\boxed{4.1}$ ~~4.2, 4.3~~ Area + Integrals

<u>Total Cost</u> : the more you make, the less it cost.

First 50 skateboards : \$40 per board
Next 75 " : \$30 " "
After 125 produced : \$25 " "

What's the total cost of producing 150 board?



Area of rectangle is lw

1st rectangle: $(50 \text{ boards})\left(\frac{\$40}{\text{board}}\right) = \$2000$

2nd rectangle: $(75 \text{ boards})\left(\frac{\$30}{\text{board}}\right) = \$2250$

3th rectangle: $(25 \text{ boards})\left(\frac{\$25}{\text{board}}\right) = \$625$

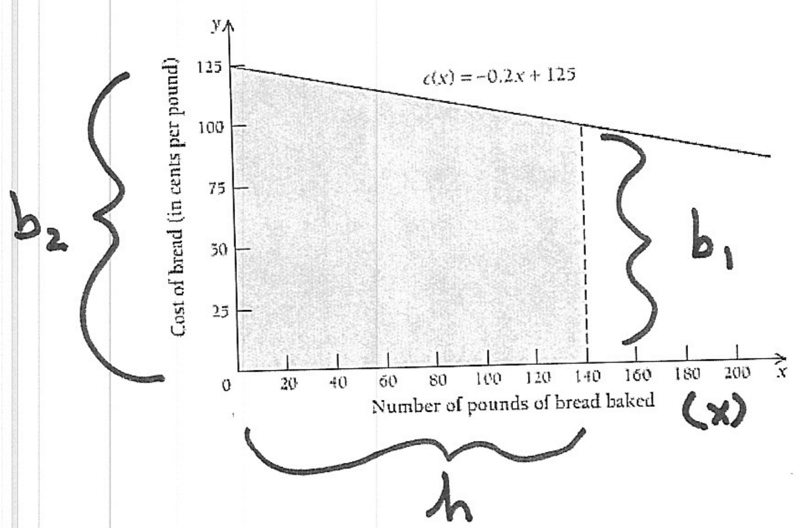
total Cost : \$4875

Often, the cost per unit changes continuously.

Say the cost per pound (¢) is

$c(x) = -0.2x + 125, \quad x < 500$

for x lbs of bread. What's the total cost for 140 lbs?



Trapezoid

$$A = h \cdot \frac{(b_1 + b_2)}{2}$$

Total Cost =

$$140 \text{ lbs} \cdot \left(\frac{97¢/\text{lb} + 125¢/\text{lb}}{2} \right) =$$

$$140 \text{ lbs} \left(\frac{111¢}{\text{lb}} \right) = 15,540¢$$

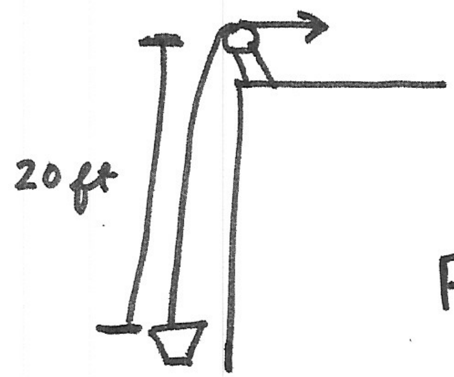
or

155.40

$W = Fd$ (Work is force x distance)

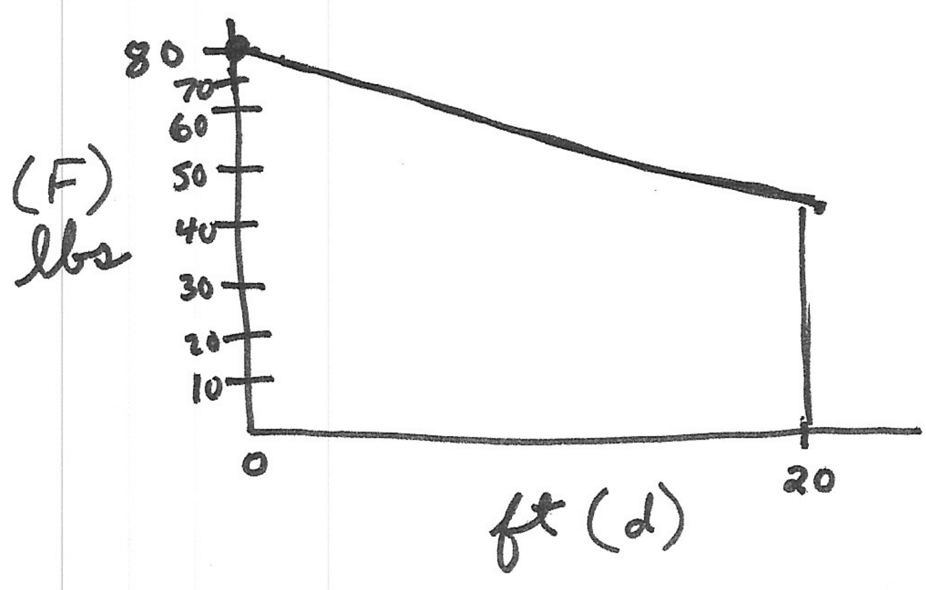
A 50-lb object is lifted from the ground into the air by pulling in 20 ft of chain that weighs 1.5 lbs/ft. How much work is done lifting this object?

F = force = weight $= 50 + 1.5(20 - d)$



d is amount of chain pull in (ft)

$F = 80 - 1.5d$



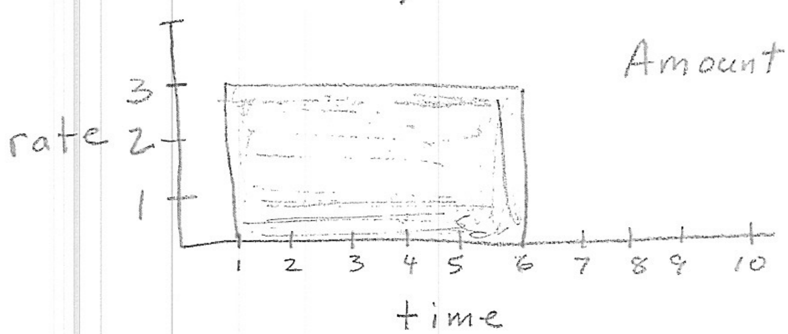
Riemann's Thought.

The rate at which sand is draining from a bag is continuously changing – the rate of change is $r(t) = \sqrt{10-t}$ (oz per second), where t is the elapsed time after the bag is cut. How much sand drained from the bag between $t = 1$ second and $t = 6$ seconds?

If $r(t)$ was constant, this is EASY. Say $r(t) = 3$ (3 oz per second)

$A = rt = (3)(6-1) = 15$ oz.

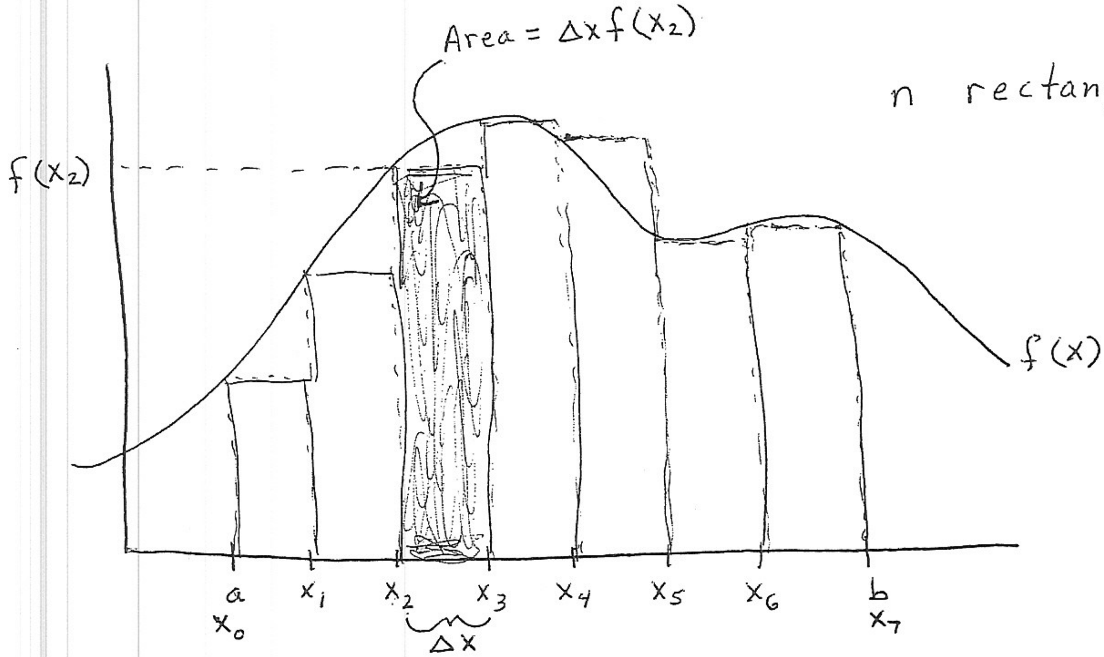
Geometrically



Amount = area of rectangle

But rate is changing!

Finding the area under a curve, $f(x) > 0$ on $[a, b]$



Area under curve is approximately equal to the sum of the areas of the rectangles.

The width of each rectangle is $\Delta x = \frac{b-a}{n}$. The length or height of each rectangle is $f(x_i)$ where $i = 0$ to $n-1$.

Area of a rectangle is $L \times W = f(x_i)\Delta x$. Approximate area under curve is $\sum_{i=0}^{n-1} f(x_i)\Delta x = \Delta x \sum_{i=0}^{n-1} f(x_i)$

Left Sum Technique

Use the left-sum technique to estimate the area under $f(x) = \sqrt{x}$ on $[1, 3]$ with 4 rectangles.

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

$x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$

$f(1) = 1$

$f(1.5) = 1.225$

$f(2) = 1.414$

$f(2.5) = 1.581$

Approximate Area is

$$\frac{1}{2}\left(1 + 1.225 + 1.414 + 1.581\right) =$$

$$\frac{1}{2}(5.22) = 2.61$$

left ~~lower~~: $\Delta x \sum_{i=0}^{n-1} f(x_i)$

right ~~upper~~: $\Delta x \sum_{i=1}^{n} f(x_i)$

Mid: $\Delta x \sum_{i=0}^{n-1} f\left(\frac{x_i + x_{i+1}}{2}\right)$

where $\Delta x = \frac{b-a}{n}$

$a = x_0 < x_1 < x_2 < \dots < x_n = b$

$x_{i+1} - x_i = \Delta x$

Right Sum Technique

Approximate Area is $\Delta x \sum_{i=1}^{n} f(x_i)$

$x_1 = 1.5, \; x_2 = 2, \; x_3 = 2.5, \; x_4 = 3$

$f(1.5) = 1.225$

$f(2) = 1.414$

$f(2.5) = 1.581$

$f(3) = 1.732$

Approximate Area is

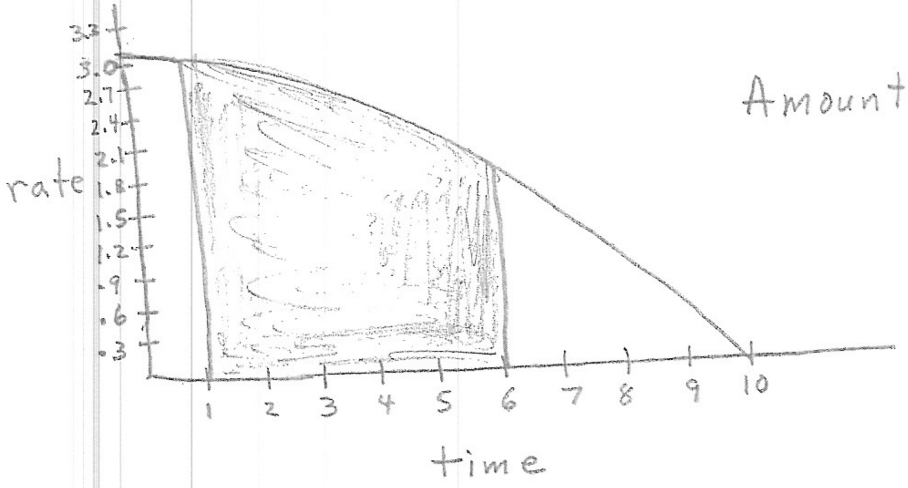
$\frac{1}{2}\left(1.225 + 1.414 + 1.581 + 1.732\right) =$

$\frac{1}{2}(5.952) = 2.976$

Avg is $\dfrac{2.61 + 2.976}{2} = 2.793$

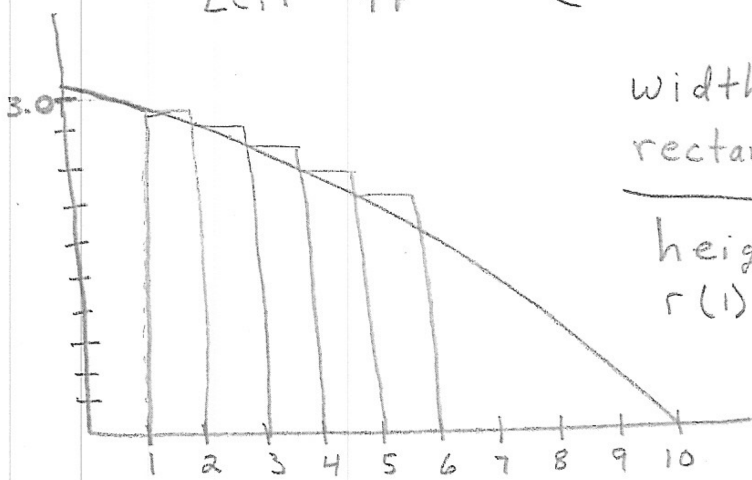
better Approximation

$r(t) = \sqrt{10 - t}$



Amount = area

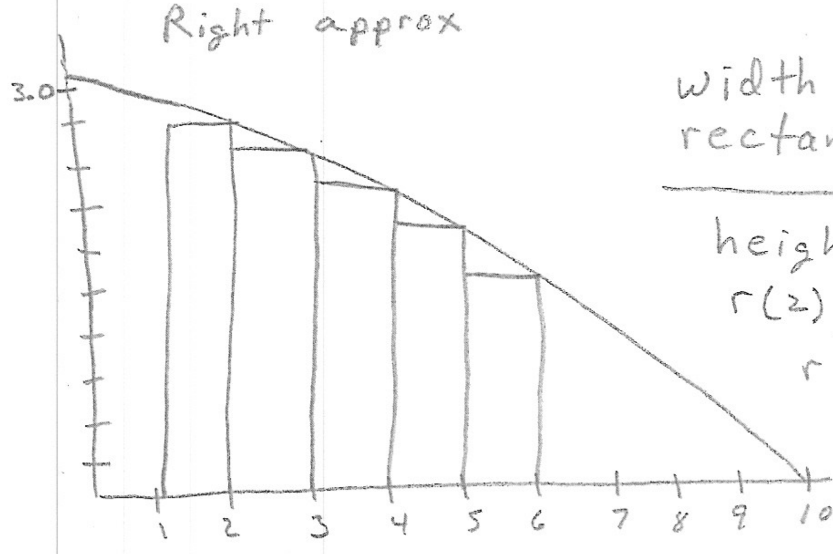
Left approx (Lower Sum)



width of each rectangle is 1 $\Big\}$ $\Delta X = 1$

heights are
r(1), r(2), r(3), r(4), r(5)

Right approx



width of each rectangle is 1 $\Big\}$ $\Delta X = 1$

heights are
r(2), r(3), r(4),
r(5), r(6)

Left Approx.

Add up the areas of the 5 rectangles

$$A = w \times l \;=\; \Delta x\, l_1 + \Delta x\, l_2 + \Delta x\, l_3 + \Delta x\, l_4 + \Delta x\, l_5$$

$$= \Delta x \sum l \;=\; 1\left[r(1) + r(2) + r(3) + r(4) + r(5)\right]$$

$$= \sqrt{9} + \sqrt{8} + \sqrt{7} + \sqrt{6} + \sqrt{5} \approx 13.16$$

Similarly, the Right Approx.

$$A = \sqrt{8} + \sqrt{7} + \sqrt{6} + \sqrt{5} + \sqrt{4} = 12.16$$

$$\text{Avg} = \frac{13.16 + 12.16}{2} = 12.66$$

Use Program on Calculator

&

Simpson's Rule