

4.2 Antiderivatives

Definition: $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Ex. An antiderivative of $f(x) = 3x^2 + 2x - 1$ is $F(x) = x^3 + x^2 - x + 7$; but so are

$$F(x) = x^3 + x^2 - x - 9$$
$$F(x) = x^3 + x^2 - x + 100$$

Definition: The general antiderivative of $f(x)$ is called the indefinite integral.

It is denoted and defined by

$$\int f(x) dx = F(x) + C \quad \text{where } C \text{ is a constant}$$

The process of finding this antiderivative is called integrating.

Basic Integration Rules on p. 406

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

if $n \neq -1$

Ex. $\int (x^3 + 4x^2 - x + 3) dx$

Ex. $\int \frac{4}{x^5} dx$

$$\text{Ex. } \int \frac{dx}{\sqrt{x}}$$

$$\text{Ex. } \int 3 dx$$

$$\text{Ex. } \int \frac{1}{x} dx$$

$$\text{Ex. } \int 4e^{7x} dx$$

$$\text{Ex. } \int \frac{7x^2 - x + 2}{x^3} dx$$

$$\text{Ex. } \int dx$$

$$\text{Ex. } \int y^2(1 - \sqrt{y}) dy$$

Ex. Find f so that $f'(x) = 3x^2$
and $f(2) = 17$

Ex. On the moon, the acceleration due to gravity is -5.3 feet per sec^2 . An astronaut shoots a bullet straight up. After 2 minutes its velocity $= \frac{dy}{dt}$ is 64 feet per second and its height is 8.683 miles.

a) What is the initial velocity of the bullet?

b) How long does it take for the bullet to hit the ground?