Find y′ (a) by applying the product rule and (b) by multiplying the factors to produce a sum of simpler terms to differentiate.

$$y = \overset{f(x)}{(5 - x^2)}\,\overset{g(x)}{(x^3 - 5x + 5)}$$

$$\frac{dy}{dx} = (5-x^2)(3x^2-5) + (x^3-5x+5)(-2x)$$

$$= 15x^2 - 25 - 3x^4 + 5x^2 - 2x^4 + 10x^2 - 10x$$

$$= -5x^4 + 30x^2 - 10x - 25$$

$$y = (5-x^2)(x^3-5x+5)$$

$$= 5x^3 - 25x + 25 - x^5 + 5x^3 - 5x^2$$

$$= -x^5 + 10x^3 - 5x^2 - 25x + 25$$

$$\frac{dy}{dx} = -5x^4 + 30x^2 - 10x - 25$$

Find the derivative.

$$y = \frac{3x^4 + 8}{x^2} = \frac{3x^4}{x^2} + \frac{8}{x^2} = 3x^2 + 8x^{-2}$$

$$\frac{dy}{dx} = 6x - 16x^{-3} \quad \text{OR} \quad 6x - \frac{16}{x^3}$$

$$y = \frac{4x^2 - 3}{7x^3 + 5} \Rightarrow$$

$$\frac{dy}{dx} = \frac{(7x^3+5)(8x) - (4x^2-3)(21x^2)}{(7x^3+5)^2}$$

$$= \frac{56x^4 + 40x - 84x^4 + 63x^2}{(7x^3+5)^2}$$

$$= \frac{-28x^4 + 63x^2 + 40x}{(7x^3+5)^2}$$

Find the derivative of the function.

$$f(s)=\frac{\sqrt{s}-3}{\sqrt{s}+1}$$

$$f(x)=\frac{\sqrt{x}-3}{\sqrt{x}+1}=\frac{x^{\frac{1}{2}}-3}{x^{1/2}+1}$$

$$f'(x)=\frac{(x^{\frac{1}{2}}+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)-(x^{\frac{1}{2}}-3)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(x^{1/2}+1)^2}$$

$$=\frac{\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\left[(x^{\frac{1}{2}}+1)-(x^{\frac{1}{2}}-3)\right]}{(x^{\frac{1}{2}}+1)^2}$$

$$=\frac{\left(\frac{1}{2}x^{-\frac{1}{2}}\right)(x^{\frac{1}{2}}+1-x^{\frac{1}{2}}+3)}{(x^{\frac{1}{2}}+1)^2}$$

$$=\frac{\left(\frac{1}{2}x^{-\frac{1}{2}}\right)(4)}{(x^{\frac{1}{2}}+1)^2}=\frac{2x^{-\frac{1}{2}}}{(x^{\frac{1}{2}}+1)^2}$$

$$=\frac{2}{x^{\frac{1}{2}}(x^{\frac{1}{2}}+1)^2}=\frac{2}{\sqrt{x}(\sqrt{x}+1)^2}$$

Differentiate the function.

$$f(x) = \frac{x}{x^{-1}+8} \cdot \frac{x}{x} = \frac{x^2}{1+8x}$$

$$f'(x) = \frac{(1+8x)(2x) - x^2(8)}{(1+8x)^2}$$

$$= \frac{2x + 16x^2 - 8x^2}{(1+8x)^2} = \frac{8x^2+2x}{(1+8x)^2}$$

Differentiate the function.

$$g(x) = 3x^{-3}(x^4 - 3x^3 + 12x - 7) = 3x - 9x^0 + 36x^{-2} - 21x^{-3}$$

$$= 3x - 9 + \underline{36x^{-2}} - \underline{21x^{-3}}$$

$$g'(x) = 3 - 72x^{-3} + 63x^{-4}$$

OR

$$= 3 - \frac{72}{x^3} + \frac{63}{x^4}$$

Differentiate.

$$F(x) = \frac{1}{4x-3} \;;\; F'(x) = \frac{(4x-3)(0) - 1(4)}{(4x-3)^2}$$

$$= \frac{-4}{(4x-3)^2}$$

Find an equation of the tangent line to the graph of $y = \dfrac{8x}{x^2+1}$ ~~at the origin and~~ at the point (1,4).

$x = 1$
$y = 4$

$$\frac{dy}{dx} = \frac{(x^2+1)(8) - 8x(2x)}{(x^2+1)^2}$$

$$= \frac{8x^2+8-16x^2}{(x^2+1)^2} = \frac{8-8x^2}{(x^2+1)^2}$$

$$m_T = \frac{dy}{dx}(1) = 0$$

$y = mx + b \Rightarrow 4 = 0x + b \Rightarrow b = 4$

$y = 0x + 4 \Rightarrow y = 4$

The cost, in dollars, of producing x jackets is given by $C(x) = 960 + 14\sqrt{x}$. Find the rate at which the average cost is changing when 576 jackets have been produced.

$$\bar{C}(x) = \text{avg cost} = \frac{C(x)}{x} = \frac{960 + 14x^{\frac{1}{2}}}{x}$$

$$= \frac{960}{x} + \frac{14x^{\frac{1}{2}}}{x}$$

$$\bar{C}(x) = 960x^{-1} + 14x^{-\frac{1}{2}}$$

$$\bar{C}'(x) = -960x^{-2} - 7x^{-3/2}$$

$$= \frac{-960}{x^2} - \frac{7}{\sqrt{x^3}}$$

$$\bar{C}'(576) = \frac{-960}{576^2} - \frac{7}{576^{3/2}}$$

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-960/576²-7/576^
1.5
      -.0033998843
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