Find all relative extrema and classify each as a maximum or minimum.

$f(x) = -125x^3 + 15x + 2$

$f'(x) = -375x^2 + 15$

$= -15(25x^2 - 1)$

$= -15(5x+1)(5x-1)$

cv: $-\frac{1}{5}$, $\frac{1}{5}$

$\left(-\frac{1}{5}, 0\right)$ + $\left(\frac{1}{5}, 4\right)$

min max

$f''(x) = -750x$

$f''\left(-\frac{1}{5}\right) = 150 > 0$ upward

$f''\left(\frac{1}{5}\right) = -150 < 0$ downward

$f''(x) = 0 \Rightarrow -750x = 0 \Rightarrow x = 0$

pt of infl: $(0, 2)$

Graph $\frac{x^2+1}{x}$ (no x- or y-intercepts)

VA: $x = 0$ (VA) $\boxed{\text{y-axis}}$

NO HA

$$y = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x}$$

oblique: $y = x$

$$y = \frac{x^2+1}{x} = x + \frac{1}{x} = x + x^{-1}$$

$$y' = 1 - x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2}{x^2} - \frac{1}{x^2}$$

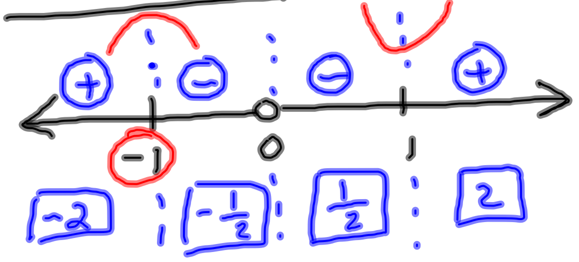
$$= \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x^2}$$

$(-1, -2)$

$(1, 2)$

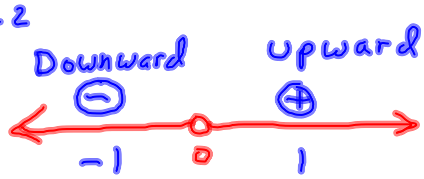
CV: $-1, 1$

pt of disc: 0

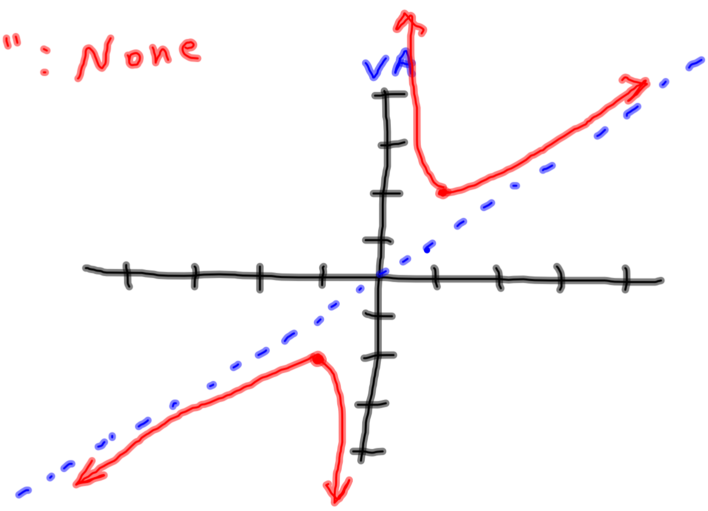


$$y' = 1 - \frac{1}{x^2} = 1 - x^{-2}$$

$$y'' = 2x^{-3} = \frac{2}{x^3}$$



CV'': None

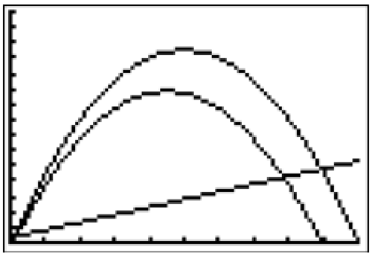


Using the same set of axes, sketch the graphs of the total-revenue, total-cost, and total-profit functions.

$R'(x) = 50 - x$

$R(x) = 50x - 0.5x^2$, $C(x) = 5x + 30$

Revenue Cost



Profit = Revenue − Cost

$$P(x) = R(x) - C(x)$$

$$= \left(50x - \tfrac{1}{2}x^2\right) - (5x + 30)$$

$$= 50x - \tfrac{1}{2}x^2 - 5x - 30$$

$$= 45x - \tfrac{1}{2}x^2 - 30$$

In some applications, the inflection point is the point of diminishing return.

Ex. A company has determined that an investment of x hundred dollars in advertisement results in higher revenue; $R(x) = 4.5\sqrt[3]{x-10} + 10$ is the daily revenue in thousands of dollars. Notice from its graph, it is an increasing function; also, beyond the inflection point, the company gets less and less "bang" for the buck. Find the point of diminishing return. The company will probably use this to determine how much they will spend in advertising.

xmin = 0	ymin = 0
xmax = 20	ymax = 20