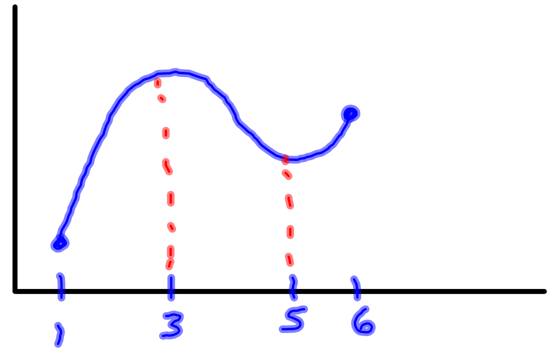
2.4 Absolute Extrema

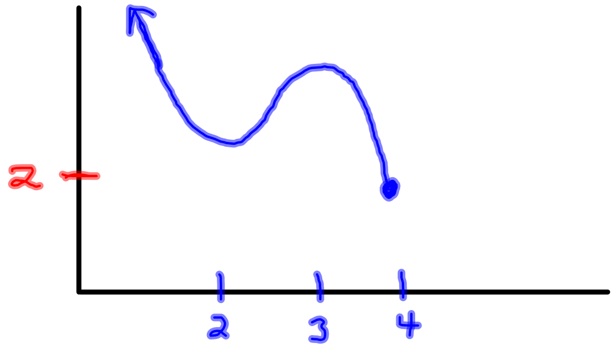
on $[1,6]$



rel max at $x=3$
rel min at $x=5$

abs max at $x=3$
abs min at $x=1$

on $(-\infty, 4]$

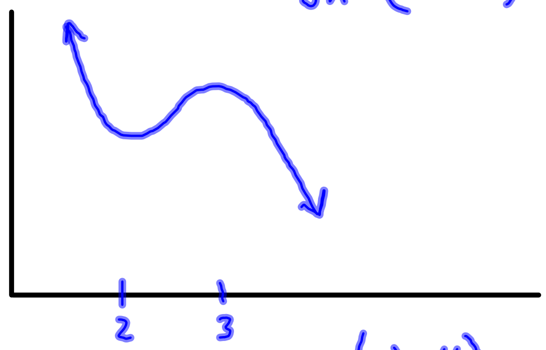


Since ∞ is not a real number there

No abs max

Abs min at $x=4$
abs min is $y=2$

on $(-\infty,\infty)$

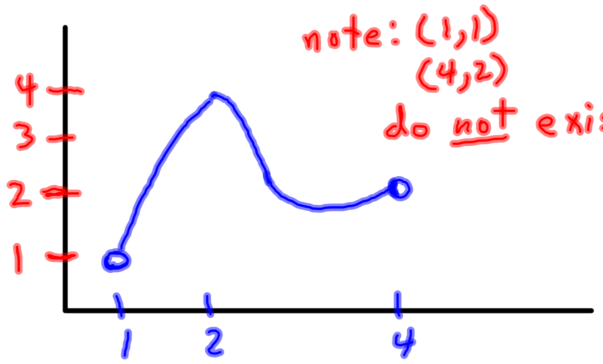


No Abs Extrema

on $(1,4)$

$\boxed{\text{No Abs Min}}$

note: $(1,1)$ $(4,2)$ do not exist



Abs Max at $x=2$

Abs Max is 4
$(2,4)$

Find the absolute maximum and minimum values of each function over the indicated interval, and indicate the x-values at which they occur.

$f(x) = x^3 - x^2 - 8x + 8; [-2, 0]$

not in $[-2,0]$

$f'(x) = 3x^2 - 2x - 8$
$= (3x+4)(x-2) \Rightarrow CV: -\frac{4}{3}, \cancel{2}$

Check: $f(-\frac{4}{3}) = \frac{392}{27}$ abs max $(-\frac{4}{3}, \frac{392}{27})$
$f(-2) = 12$
$f(0) = 8$ abs min $(0, 8)$

```
(-4/3)^3-(-4/3)²
-8(-4/3)+8
         14.51851852
Ans►Frac
              392/27
```

```
Y₁(-4/3)
         14.51851852
Y₁(-2)
                  12
Y₁(0)
                   8
```

$f(x) = x^4 - 18x^2 + 5; [-5, 5]$

$f'(x) = 4x^3 - 36x$
$= 4x(x^2 - 9) = 4x(x+3)(x-3)$
$cv: 0, -3, 3$

$f(-5)$
$f(-3)$
$f(0)$
$f(3)$
$f(5)$

X	Y₁	
-5	180	
-3	-76	
0	5	
3	-76	
5	180	

X=

abs max
$(-5, 180)$
$(5, 180)$

abs min
$(-3, -76)$
$(3, 76)$

$f(x) = (x+8)^{1/3}; \ [-16, 19]$

$f'(x) = \frac{1}{3}(x+8)^{-2/3}(1) = \dfrac{1}{3(x+8)^{2/3}}$

CV: -8

$f(-16) =$

$f(-8) =$

$f(19) =$

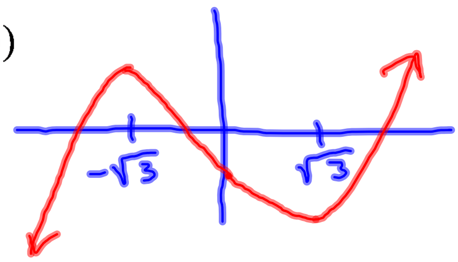
X	Y1	
-16	-2	
-8	0	
19	3	

X=

abs min: $(-16, -2)$

abs max: $(19, 3)$

$f(x) = \frac{2}{3}x^3 - 6x + 3$ on the interval $(-\infty, \infty)$

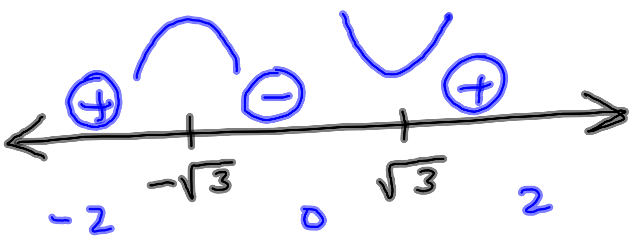


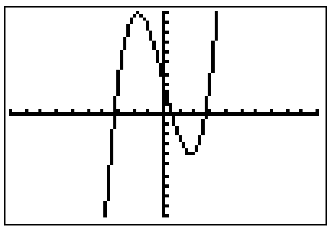
$f'(x) = 2x^2 - 6$

$= 2(x^2 - 3)$

$f'(x) = 0$ if $x^2 - 3 = 0 \Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$





No Abs Min

No Abs Max

$f(x) = x^4 - 4x^2$ on the interval $(-\infty, \infty)$

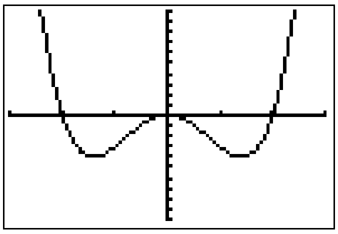
$f'(x) = 4x^3 - 8x$

$= 4x(x^2 - 2)$

$x^2 - 2 = 0 \Rightarrow$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

cv: $0, -\sqrt{2}, \sqrt{2}$



No Abs Max

Abs Min at

$(-\sqrt{2}, -4)$ + $(\sqrt{2}, -4)$

$f(x) = x^2 + \dfrac{240}{x}; \quad (0, \infty)$

$f(\sqrt[3]{120}) = 120^{2/3} + \dfrac{240}{120^{1/3}}$

$f(x) = x^2 + 240x^{-1}$

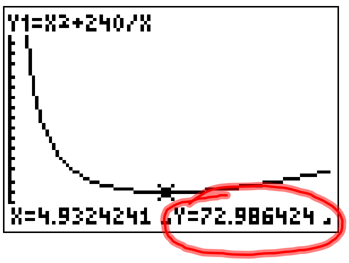
$f'(x) = 2x - 240x^{-2}$

$= \dfrac{2x}{1} \cdot \dfrac{x^2}{x^2} - \dfrac{240}{x^2} = \dfrac{2x^3 - 240}{x^2}$

$= \dfrac{2(x^3 - 120)}{x^2}$

$x^3 - 120 = 0 \Rightarrow x^3 = 120$

$\Rightarrow x = \sqrt[3]{120}$



no abs max

abs min at $(\sqrt[3]{120},$ see above$)$

The total-cost, $C(x)$, and total-revenue, $R(x)$, functions for producing x items are shown below, where $0 \le x \le 300$. $[0, 300]$

$$C(x) = 5200 + 500x \text{ and } R(x) = -\frac{1}{2}x^2 + 600x$$

a) Find the total-profit function $P(x)$.
b) Find the number of items, x, for which the total profit is a maximum.

a) $P(x) = R(x) - C(x)$

$= -\frac{1}{2}x^2 + 600x - (5200 + 500x)$

$= -\frac{1}{2}x^2 + 600x - 5200 - 500x$

$= -\frac{1}{2}x^2 + 100x - 5200$

b) $P'(x) = -x + 100$

$-x + 100 = 0 \Rightarrow x = 100$

$$\frac{-b}{2a}$$

$$f(x) = ax^2 + bx + c$$

$$f'(x) = 2ax + b = 0$$

$$2ax = -b$$

$$x = \frac{-b}{2a}$$