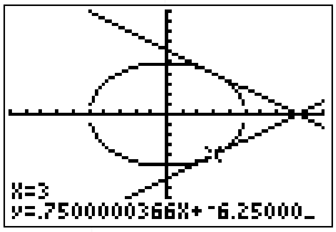
2.7 Implicit Differentiation

This technique is best when dealing with equations that are not functions; here, y is usually difficult to solve for.

Ex. Find the slope of the tangent line to the curve $x^2+y^2=25$ at the point $(3,4)$.

not a function

NOTE $\left\{ \begin{array}{l} x^2+y^2=25 \text{ is a circle with center} \\ (0,0) \text{ and radius } 5. \end{array} \right.$



Solve for y: $y^2 = 25 - x^2$

$$y = \pm\sqrt{25-x^2}$$

$$= \pm(25-x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \pm\frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \frac{\pm x}{\sqrt{25-x^2}} \qquad \text{let } x=3$$

$$m = \frac{\pm 3}{\sqrt{25-9}} = \frac{\pm 3}{\sqrt{16}} = \frac{\pm 3}{4} \quad \Bigg\} \text{ Is it } \frac{3}{4} \text{ or } -\frac{3}{4}\text{?}$$

$\frac{dy}{dx}$ is the derivative of the function y with respect to the variable x.

Implicit means hidden

$$x^2 + y^2 = 25$$

y is hidden

$y = \pm\sqrt{25 - x^2}$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

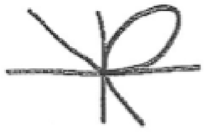
$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

slope at $(3,4)$ is

$$m = \frac{-x}{y} = \frac{-3}{4}.$$

Find the equation of the tangent line at (2, 4) for the curve

$x^3 + y^3 = 9x \cdot y$ [Folium of Descartes]



$$3x^2 + 3y^2 \frac{dy}{dx} = 9x \frac{dy}{dx} + 9y$$

$$3y^2 \frac{dy}{dx} - 9x \frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 - 9x\right) = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x} = \frac{3(3y - x^2)}{3(y^2 - 3x)}$$

$$= \boxed{\frac{3y - x^2}{y^2 - 3x}}$$

$$m = \frac{3(4) - 2^2}{4^2 - 3(2)} = \frac{12 - 4}{16 - 6} = \frac{8}{10} = \frac{4}{5}$$

$$y - 4 = \frac{4}{5}(x - 2) \Rightarrow y - 4 = \frac{4}{5}x - \frac{8}{5}$$

$$+\frac{20}{5} \qquad\qquad +\frac{20}{5}$$

$$y = \frac{4}{5}x + \frac{12}{5}$$