$$\underline{y^2} + \underline{x^3} = \underline{\overset{y^{1/2}}{\sqrt{y}}} + \underline{8} \; ; \text{ find } \frac{dy}{dx}$$

$$2y \cdot \frac{dy}{dx} + 3x^2 = \frac{1}{2} y^{-\frac{1}{2}} \cdot \frac{dy}{dx} + 0$$

$$2\left(2y \frac{dy}{dx} - \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx}\right) = 2\left(-3x^2\right)$$

$$4y \frac{dy}{dx} - y^{-\frac{1}{2}} \frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx}\left(4y - y^{-\frac{1}{2}}\right) = -6x^2$$

$$\frac{dy}{dx} = \frac{-6x^2}{4y - y^{-\frac{1}{2}}} \cdot \frac{y^{\frac{1}{2}}}{y^{\frac{1}{2}}} \overset{\boxed{\text{OR}}}{=} \frac{-6x^2 y^{\frac{1}{2}}}{4y^{3/2} - 1}$$

$$\underline{4y^6} - \underline{8x} = \underline{y} - 1 \; ; \text{ find } \frac{dy}{dx}$$

$$24y^5 \cdot \frac{dy}{dx} - 8 = \frac{dy}{dx}$$

$$24y^5 \cdot \frac{dy}{dx} - \frac{dy}{dx} = 8$$

$$\frac{dy}{dx}\left(24y^5 - 1\right) = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{24y^5 - 1}$$

$$(y)^4 - 3x^2 = y^2 - 2 \text{ ; find } \frac{dy}{dx}$$

$$4y^3 \cdot \frac{dy}{dx} - 6x = 2y\frac{dy}{dx}$$

$$4y^3\frac{dy}{dx} - 2y\frac{dy}{dx} = 6x$$

$$\frac{dy}{dx}(4y^3 - 2y) = 6x \Rightarrow$$

$$\frac{dy}{dx} = \frac{6x}{4y^3 - 2y} = \frac{2(3x)}{2(2y^3 - y)} = \frac{3x}{2y^3 - y}$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$x^2 \cdot y^5 + 7x = y - 8$$

$$x^2\left(5y^4\frac{dy}{dx}\right) + y^5(2x) + 7 = \frac{dy}{dx}$$

$$5x^2y^4\frac{dy}{dx} + 2xy^5 + 7 = \frac{dy}{dx}$$

$$5x^2y^4\frac{dy}{dx} - \frac{dy}{dx} = -2xy^5 - 7$$

$$\frac{dy}{dx}(5x^2y^4 - 1) = -2xy^5 - 7 \Rightarrow$$

$$\frac{dy}{dx} = \frac{-2xy^5 - 7}{5x^2y^4 - 1} \cdot \frac{(-1)}{(-1)} = \frac{2xy^5 + 7}{1 - 5x^2y^4}$$

Related Rates: $\frac{df}{dt}$

Ex. Let $8y^3 + x^2 = 1$;

$\frac{dx}{dt} = 2$, $x = 3$, $y = -1$.

Find $\frac{dy}{dt}$.

Solution: Here, x and y are functions of time.

$$\left.\begin{aligned} x(t) &= ? \\ y(t) &= ? \end{aligned}\right\} \text{what they are is hidden}$$

We will use implicit differentiation.

$8y^3 + x^2 = 1 \Rightarrow$

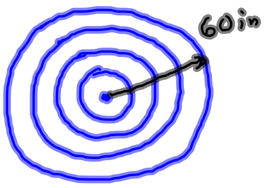
$24y^2 \cdot \frac{dy}{dt} + 2x\frac{dx}{dt} = 0$; plug in values

$24(-1)^2 \cdot \frac{dy}{dt} + 2(3)(2) = 0 \Rightarrow$

$24\frac{dy}{dt} + 12 = 0 \Rightarrow \frac{dy}{dt} = -\frac{1}{2}$

A pebble is dropped into a pond and concentric circles ~~centers~~ form. If the radius is increasing at 8 inches per second, how fast is the area of the outer circle changing when the radius of the outer circle is 60 inches?

$$\rightarrow \quad A = \pi r^2$$



$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\left\{ \begin{array}{l} \dfrac{dr}{dt} = 8 \text{ in/sec} \\ r = 60 \text{ inches} \\ \dfrac{dA}{dt} = ? \end{array} \right.$$

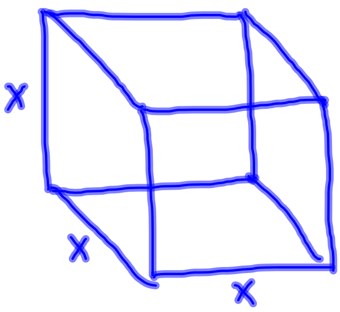
$$= 2\pi (60 \text{ in})(8 \text{ in/sec})$$

$$= 960\pi \text{ in}^2/\text{sec}$$

OR

$$\approx 3016 \text{ in}^2/\text{sec}$$

All edges of an ice cube are melting at a rate of 5 millimeters per minute. How fast is the volume of the cube changing when the lengths of the edges are 1 centimeter. (1 cm = 10 mm)



$\frac{dx}{dt} = -5 \text{ mm/min}$

$x = 10 \text{ mm}$

$\frac{dV}{dt} = ?$

$$V = x^3$$

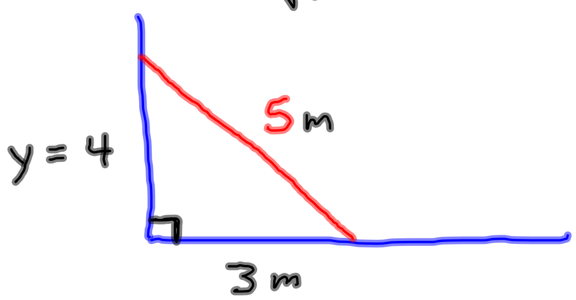
$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3(10 \text{ mm})^2(-5 \text{ mm/min})$$

$$= -1500 \text{ mm}^3/\text{min}$$

$$= -1.5 \text{ cm}^3/\text{min}$$

A 5-meter ladder resting on a wall begins to slip. The top of the ladder is falling at a rate of 1.5 meters per second when the bottom of the ladder is 3 meters from the wall. At this time, how fast is the bottom of the ladder moving away from the wall?

$$\sqrt{5^2 - 3^2} = \sqrt{16} = 4$$



$$\frac{dy}{dt} = -1.5 \text{ m/sec}$$

$$x = 3$$

$$\frac{dx}{dt} = ?$$

$$x^2 + y^2 = 25$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

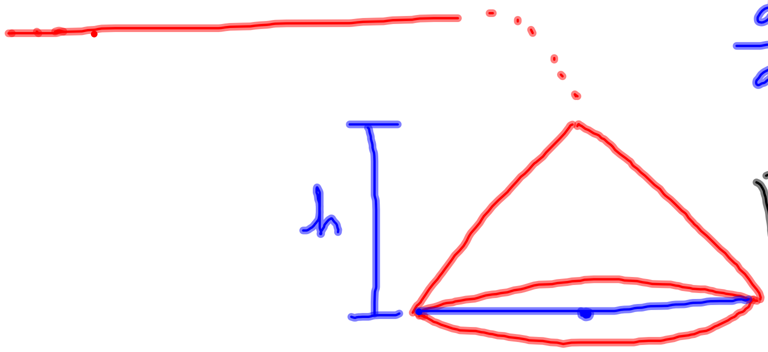
$$\Rightarrow 3\frac{dx}{dt} + 4(-1.5) = 0$$

$$\Rightarrow 3\frac{dx}{dt} - 6 = 0$$

$$\Rightarrow 3\frac{dx}{dt} = 6$$

$$\Rightarrow \frac{dx}{dt} = 2 \text{ m/sec}$$

At a sand and gravel plant, sand is falling off a conveyor and onto a pile at a rate of 10 cubic feet per minute. The diamter of the base is three times its height. At what rate is the height changing when it is 15 feet high?



$$\frac{dV}{dt} = 10\ ft^3/min$$

$$h = 15\ ft$$

$$\boxed{\frac{dh}{dt} = ?}$$

$$d = 3h$$

$$2r = 3h \Rightarrow r = \frac{3h}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h = \frac{1}{3}\pi\left(\frac{9h^2}{4}\right)h$$

$$V = \frac{3}{4}\pi h^3 \Rightarrow$$

$$\frac{dV}{dt} = \frac{9}{4}\pi h^2 \cdot \frac{dh}{dt} \Rightarrow$$

$$10 = \frac{9}{4}\pi (15)^2 \cdot \frac{dh}{dt} \Rightarrow$$

$$10 = \frac{2025\pi}{4} \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{10}{2025\pi/4} \Rightarrow$$

$$\frac{dh}{dt} = \frac{40}{2025\pi} = \frac{8}{405\pi}\ ft/min \approx .006\ ft/min$$