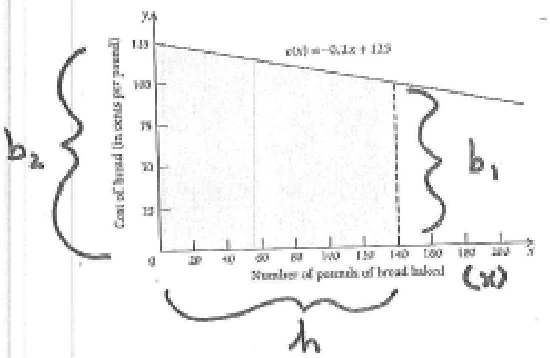
4.2 Area and Integrals

Say the cost per pound (¢) is
$c'(x) = -0.2x + 125, \quad x < 500$ [Marginal Cost]
for x lbs of bread. What's the total cost for 140 lbs?



Trapezoid

$$A = h \cdot \frac{(b_1 + b_2)}{2}$$

Total Cost =

$$140 \text{ lbs} \cdot \left(\frac{97¢/lb + 125¢/lb}{2} \right) =$$

$$140 \text{ lbs} \left(\frac{111¢}{lb} \right) = 15{,}540¢$$

or

155.40

$$F(x) = \int c'(x)\,dx = \int (-0.2x + 125)\,dx$$

$$= -0.1x^2 + 125x + ¢$$

$$F(140) = -0.1(140)^2 + 125(140) + ¢$$

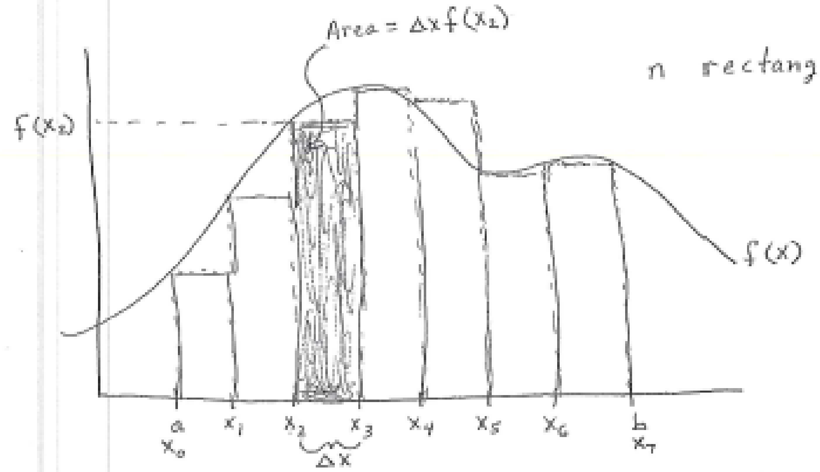
$$= 15{,}540 + ¢$$

$$F(0) = ¢$$

$$\text{Total Cost} = F(140) - F(0) = 15{,}540¢$$

$$= \$155.40$$

Finding the area under a curve, $f(x) > 0$ on $[a, b]$



n rectangles

Area under curve is approximately equal to the sum of the areas of the rectangles.
The width of each rectangle is $\Delta x = \frac{b-a}{n}$. The length or height of each rectangle is $f(x_i)$ where $i = 0$ to $n-1$.
Area of a rectangle is $L \times W = f(x_i)\Delta x$. Approximate area under curve is $\sum_{i=0}^{n-1} f(x_i)\Delta x = \Delta x \sum_{i=0}^{n-1} f(x_i)$

Left Sum Technique

Riemann's Sum

Use the left-sum technique to estimate the area under $f(x) = \sqrt{x}$ on $[1, 3]$ with 4 rectangles.

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

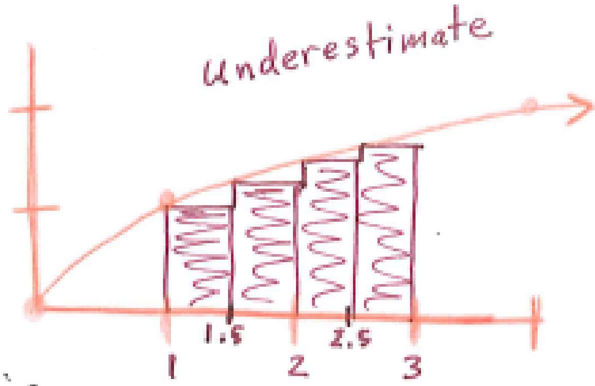
$x_0 = 1,\ x_1 = 1.5,\ x_2 = 2,\ x_3 = 2.5$

$f(1) = 1$

$f(1.5) = 1.225$

$f(2) = 1.414$

$f(2.5) = 1.581$



Approximate Area is

$$\frac{1}{2}\left(1 + 1.225 + 1.414 + 1.581\right) =$$

$$\frac{1}{2}(5.22) = 2.61$$

Right Sum Technique

Approximate Area is $\Delta x \sum_{i=1}^{n} f(x_i)$

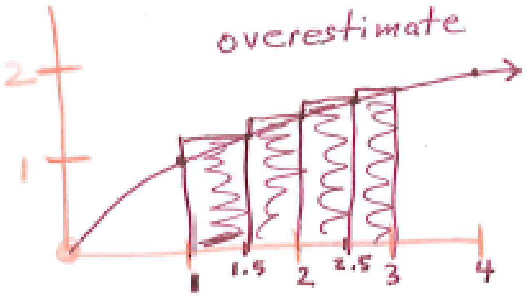
$x_1 = 1.5,\ x_2 = 2,\ x_3 = 2.5,\ x_4 = 3$

$f(1.5) = 1.225$

$f(2) = 1.414$

$f(2.5) = 1.581$

$f(3) = 1.732$



Approximate Area is

$$\frac{1}{2}(1.225 + 1.414 + 1.581 + 1.732) =$$

$$= \frac{1}{2}(5.952) = 2.976$$

Avg is $\dfrac{2.61 + 2.976}{2} = 2.793$

better Approximation

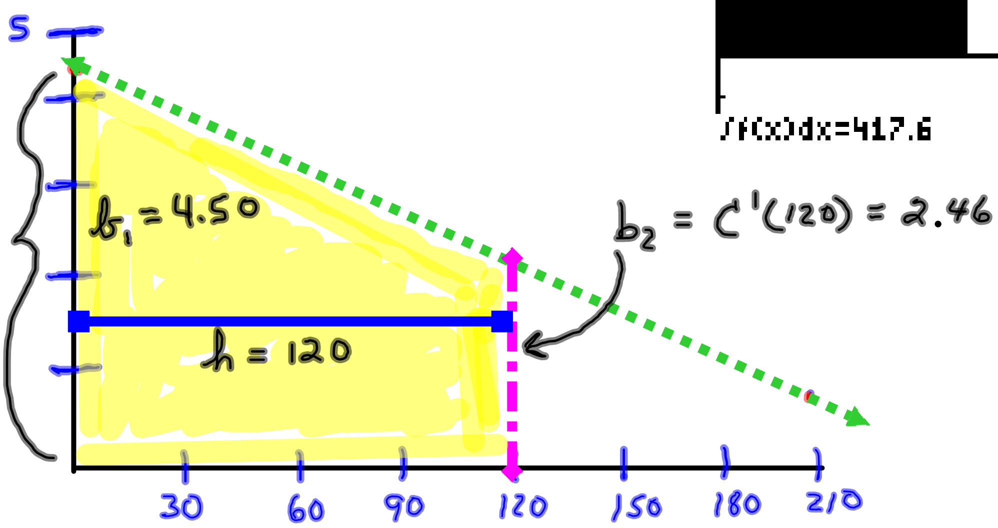
$$F(x) = \int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx = \frac{x^{3/2}}{3/2} + c$$

$$= \frac{2}{3} x^{3/2} + C$$

$$\left.\begin{array}{l} F(3) = 3.464 + C \\ F(1) = 0.667 + C \end{array}\right\} F(3) - F(1) = \boxed{2.797}$$

A coffee company has found that the marginal cost, in dollars per pound, of the coffee it roasts is represented by the function below, where x is the number of pounds of coffee roasted. Find the total cost of roasting 120 lb of coffee, disregarding any fixed costs.

$$C'(x) = -0.017x + 4.50, \text{ for } x \leq 200$$



$$A_{trapezoid} = \frac{h}{2}(b_1 + b_2) = 60(4.50 + 2.46)$$
$$= 417.60$$