

Evaluate.

$$\begin{aligned}\int_2^5 (5x^3 + 9) dx &= \left. \frac{5x^4}{4} + 9x \right|_2^5 = \\ &= \left[\frac{5(5)^4}{4} + 9(5) \right] - \left[\frac{5(2)^4}{4} + 9(2) \right] = \\ &= \left[\frac{3125}{4} + \frac{180}{4} \right] - \left[\frac{80}{4} + \frac{72}{4} \right] = \\ &= \frac{3125 + 180 - 80 - 72}{4} = \frac{3153}{4}\end{aligned}$$

Evaluate.

$$\begin{aligned}\int_1^9 (\sqrt{x} - 2) dx &= \int_1^9 (x^{\frac{1}{2}} - 2) dx = \\ &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2x \right]_1^9 = \left[\frac{2}{3} x^{\frac{3}{2}} - 2x \right]_1^9 \\ &= \left[\frac{2}{3} (27) - 18 \right] - \left[\frac{2}{3} (1) - 2 \right] = \\ &= 0 - \left(\frac{2}{3} - \frac{6}{3} \right) = 0 - \left(-\frac{4}{3} \right) \\ &= \frac{4}{3}\end{aligned}$$

Find $s(t)$, where $s(t)$ represents the position function and $v(t)$ represents the velocity function.

$$v(t) = 12t^2, \underline{s(0) = 6}$$

$$\begin{aligned} \Delta(t) &= \int 12t^2 dt = \frac{12t^3}{3} + C \\ &= 4t^3 + C \end{aligned}$$

$$\Delta(0) = 6 \Rightarrow \Delta(0) = 4(0)^3 + C \Rightarrow 6 = C$$

$$\Delta(t) = 4t^3 + 6$$

Find $s(t)$, where $s(t)$ represents the position function, $v(t)$ represents the velocity function, and $a(t)$ represents the acceleration function.

$$\underline{a(t) = -6t + 4}, \text{ with } \underline{v(0) = 9} \text{ and } \underline{s(0) = 6}$$

$$v(t) = \int (-6t + 4) dt = -\frac{6t^2}{2} + 4t + C$$

$$v(0) = C \Rightarrow v(0) = 9 \Rightarrow C = 9$$

$$v(t) = -3t^2 + 4t + 9$$

$$\begin{aligned} \Delta(t) &= \int (-3t^2 + 4t + 9) dt = -\frac{3t^3}{3} + \frac{4t^2}{2} + 9t + k \\ &= -t^3 + 2t^2 + 9t + k \end{aligned}$$

$$\Delta(0) = k; \Delta(0) = 6 \Rightarrow k = 6$$

$$\Delta(t) = -t^3 + 2t^2 + 9t + 6$$

If a race car increases speed at constant acceleration so that it goes from 0 to 120 miles per hour in 5 seconds, how far has the car traveled during the 5 seconds?

$$\begin{aligned}
 a(t) &= \frac{v(5) - v(0)}{5 - 0} = \frac{(120 - 0) \text{ mph}}{(5) \text{ sec}} \\
 &= \frac{120 \cancel{\text{ mi}}}{\cancel{\text{ hr}} \cdot \text{sec}} \cdot \frac{1 \cancel{\text{ hr}}}{3600 \text{ sec}} \cdot \frac{5280 \cancel{\text{ ft}}}{1 \cancel{\text{ mi}}} \\
 &= \frac{176 \text{ ft}}{\text{sec}^2} \quad \begin{array}{l} 120 \cdot 5280 / 3600 \\ \blacksquare \quad 176 \end{array}
 \end{aligned}$$

$$v(t) = \int 176 \, dt = 176t + C \quad \begin{array}{l} \rightarrow 0 \\ \text{since } v(0) = 0 \end{array}$$

$$v(t) = 176t$$

$$\begin{aligned}
 \Delta(t) &= \int 176t \, dt = \frac{176t^2}{2} + k \\
 &= 88t^2 + k \quad \rightarrow 0
 \end{aligned}$$

since $s(0) = 0$

$$\Delta(t) = 88t^2$$

$$\Delta(5) = 88(5)^2 = \underline{2200 \text{ feet}}$$

A particle is released as part of an experiment. Its speed t seconds after release is given by $v(t) = -0.6t^2 + 4t$, where $v(t)$ is in meters per second.

- a) How far does the particle travel during the first 2 sec?
b) How far does it travel during the second 2 sec?

$$\Delta(t) = \int (-0.6t^2 + 4t) dt$$
$$= \frac{-0.6t^3}{3} + \frac{4t^2}{2} + C$$

$$\Delta(t) = -0.2t^3 + 2t^2$$

$$\begin{aligned} \text{a) } \Delta(2) - \Delta(0) &= -0.2(2)^3 + 2(2)^2 \\ &= -1.6 + 8 = \boxed{6.4 \text{ m}} \end{aligned}$$

$$\begin{aligned} \text{b) } \Delta(4) - \Delta(2) &= \\ \Delta(4) &= -0.2(4)^3 + 2(4)^2 \\ &= -0.2(64) + 32 \\ &= -12.8 + 32 = 19.2 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta(4) - \Delta(2) &= (19.2 - 6.4) \text{ m} \\ &= \boxed{12.8 \text{ m}} \end{aligned}$$

Evaluate.

$$\int_4^8 \frac{x^2 - 1}{x - 1} dx = \int_4^8 \frac{(x+1)(x-1)}{x-1} dx =$$
$$\int_4^8 (x+1) dx = \left. \frac{x^2}{2} + x \right|_4^8 =$$
$$\left(\frac{64}{2} + 8 \right) - \left(\frac{16}{2} + 4 \right) = 32 + 8 - 8 - 4$$
$$= \boxed{28}$$

Evaluate.

$$\int_4^{16} \frac{7t+4}{5\sqrt{t}} dt = \frac{1}{5} \int_4^{16} \frac{7t+4}{t^{1/2}} dt =$$
$$\frac{1}{5} \int_4^{16} \frac{7t}{t^{1/2}} dt + \frac{1}{5} \int_4^{16} 4t^{-1/2} dt =$$
$$\frac{7}{5} \int_4^{16} t^{1/2} dt + \frac{4}{5} \int_4^{16} t^{-1/2} dt =$$
$$\frac{7}{5} \int_4^{16} t^{1/2} dt = \frac{7}{5} \left(\frac{t^{3/2}}{3/2} \right) \Big|_4^{16} = \frac{7}{5} \left(\frac{2}{3} t^{3/2} \right) \Big|_4^{16}$$
$$= \frac{14}{15} t^{3/2} \Big|_4^{16} = \frac{14}{15} (64 - 8)$$
$$= \frac{14}{15} (56) = \underline{\underline{\frac{784}{15}}}$$

$$\frac{4}{5} \int_4^{16} t^{-1/2} dt = \frac{4}{5} \left(\frac{t^{1/2}}{1/2} \right) \Big|_4^{16}$$
$$= \frac{4}{5} (2t^{1/2}) \Big|_4^{16}$$
$$= \frac{8}{5} (t^{1/2}) \Big|_4^{16} = \frac{8}{5} (4 - 2)$$
$$= \underline{\underline{\frac{16}{5}}}$$

$$\frac{784}{15} + \frac{48}{15} = \boxed{\frac{832}{15}}$$