

$$\int_{x=3}^{x=7} \frac{4x}{1+x^2} dx \quad \text{let } u = 1+x^2$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\int_{u=10}^{u=50} \frac{4x}{u} \cdot \frac{du}{2x} = 2 \int_{10}^{50} \frac{du}{u}$$

$$= 2 \left[\ln u \right]_{10}^{50}$$

$$= 2 \left[\ln 50 - \ln 10 \right]$$

$$= 2 \ln 5 = \ln 25$$

$$\int x^3 e^{x^4} dx \quad \text{let } u = x^4$$

$$\frac{du}{dx} = 4x^3$$

$$dx = \frac{du}{4x^3}$$

$$\int x^3 e^u \cdot \frac{du}{4x^3} = \frac{1}{4} \int e^u du$$

$$= \frac{1}{4} e^u + c$$

$$= \frac{1}{4} e^{x^4} + c$$

$$\int_{x=0}^{x=\ln 4} \frac{e^x}{(1+e^x)^3} dx$$

$$\text{let } u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x}$$

$$\begin{aligned} \int_2^5 \frac{e^x}{u^3} \cdot \frac{du}{e^x} &= \int_2^5 u^{-3} du \\ &= \left. \frac{u^{-2}}{-2} \right|_2^5 = \left. -\frac{1}{2u^2} \right|_2^5 \\ &= -\frac{1}{50} - \left(-\frac{1}{8}\right) \\ &= -\frac{4}{200} + \frac{25}{200} = \frac{21}{200} \end{aligned}$$

$$\int \frac{e^x e^x}{(e^{-x} + 1)e^x} dx = \int \frac{e^{2x}}{1 + e^x} dx =$$

$$\text{let } u = 1 + e^x \Rightarrow e^x = u - 1 \quad \left\{ \frac{(u-1)^2}{u} \cdot \frac{du}{u-1} \right.$$

$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x} = \frac{du}{u-1}$$

$$\left\{ \frac{u-1}{u} du \right.$$

$$\rightarrow \int \left(\frac{u}{u} - \frac{1}{u} \right) du = \int \left(1 - \frac{1}{u} \right) du$$

$$= u - \ln|u| + C$$

$$= (1 + e^x) - \ln(1 + e^x) + C$$