4.5 u-substitution

$$\int x^3 (x^4+5)^9 \, dx$$

let $u = x^4 + 5$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 \, dx$

$dx = \frac{du}{4x^3}$

$$\int \cancel{x^3} \cdot u^9 \cdot \frac{du}{4\cancel{x^3}} =$$

$$\frac{1}{4}\int u^9 \, du = \frac{1}{4} \cdot \frac{u^{10}}{10} + C$$

$$= \frac{u^{10}}{40} + C = \frac{(x^4+5)^{10}}{40} + C$$

$$\int x^2 (x^3-1)^4 \, dx$$

let $u = x^3 - 1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$\int \cancel{x^2} \cdot u^4 \cdot \frac{du}{3\cancel{x^2}} =$$

$$\frac{1}{3}\int u^4 \, du = \frac{1}{3}\left[\frac{u^5}{5}\right] + C$$

$$= \frac{u^5}{15} + C = \frac{(x^3-1)^5}{15} + C$$

$$\int_{x=3}^{x=\sqrt{14}} \frac{2x}{\sqrt{x^2-5}}\,dx$$

let $u = x^2 - 5$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\int_4^9 \frac{\cancel{2x}}{u^{1/2}} \cdot \frac{du}{\cancel{2x}} = \int_4^9 u^{-\frac{1}{2}}\,du =$$

$$\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_4^9 = 2\left[\sqrt{u}\right]_4^9 = 2(3-2) = 2$$

$$\int x\underbrace{(x-1)^4}_{u^4}\,dx$$

$\Rightarrow x = u+1$

let $\underline{u = x-1}$

$\frac{du}{dx} = 1$

$dx = du$

$$\int u^4(u+1)\,du =$$

$$\int (u^5 + u^4)\,du = \frac{u^6}{6} + \frac{u^5}{5} + C$$

$$= \frac{(x-1)^6}{6} + \frac{(x-1)^5}{5} + C$$

$$\int \frac{8}{3+2x}\,dx$$

$u = 3+2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$\int \frac{8}{u}\cdot\frac{du}{2} = \int \frac{4}{u}\,du = 4\int\frac{du}{u}$$

$$= 4\ln|u| + c = 4\ln|3+2x| + c$$

$$\int \frac{(\ln x)^7}{x}\,dx$$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$$\int \frac{u^7}{\cancel{x}}\cdot\cancel{x}\,du = \int u^7\,du = \frac{u^8}{8} + c$$

$$= \frac{(\ln x)^8}{8} + c$$

$$\int_{x=3}^{x=7} \frac{4x}{1+x^2}\,dx$$

let $u = 1 + x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\int_{u=10}^{u=50} \frac{4x}{u} \cdot \frac{du}{2x} = 2\int_{10}^{50} \frac{du}{u}$$

$$= 2\Big[\ln u\Big]_{10}^{50}$$

$$2\Big[\ln 50 - \ln 10\Big]$$

$$= 2\ln 5 = \ln 25$$

$$\int x^3 e^{x^4}\,dx$$

$$\int_0^{\ln 4} \frac{e^x}{(1+e^x)^3}$$

$$\int \frac{e^x}{e^{-x}+1}\,dx$$