Item 2: (1.2, p. ~~121~~ 118)

$x \neq 5$
$x \to 5$

20. $\displaystyle\lim_{x \to 5} \frac{x^2 - 25}{x - 5} =$

$\displaystyle\lim_{x \to 5} \frac{(x+5)(x-5)}{x-5} =$

$\displaystyle\lim_{x \to 5} x + 5 = 10$

note: if $f(x) = \dfrac{x^2 - 25}{x - 5}$

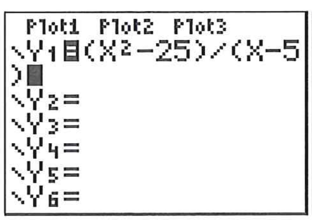
$f(5) = \dfrac{0}{0}$

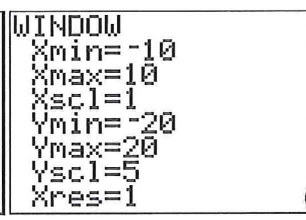
$= \text{undefined}$

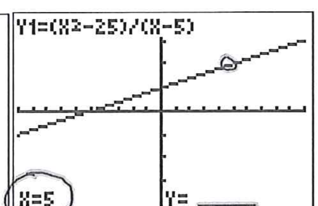
$\dfrac{0}{0}$ is indeterminate.

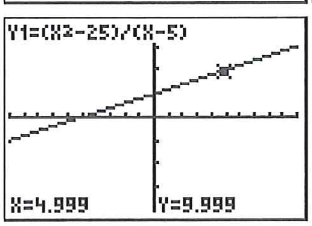
Note: $x = 5$ is <u>NOT</u> a vertical asymptote

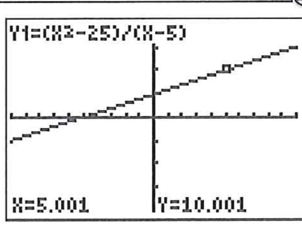
There is a <u>hole</u> at $x = 5$











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Item 4: (1.3, p. ~~133~~)

41. Average velocity. In t seconds, an object dropped from a certain height will fall $s(t)$ feet, where

$s(t) = 16t^2.$

a) Find $s(5) - s(3)$.

b) What is the average rate of change of distance with respect to time during the period from 3 to 5 sec? This is also *average velocity.*

Avg rate of change on $[a, b]$

$$\frac{f(b) - f(a)}{b - a}$$

Instantaneous rate of change at a

$$f'(a)$$

a) $s(5) = 16(5)^2 = 400$

$s(3) = 16(3)^2 = 144$

$s(5) - s(3) = 256 \text{ ft}$

b) velocity = avg rate of change of displacement

$$= \frac{s(5) - s(3)}{5 - 3} = \frac{256 \text{ ft}}{2 \text{ sec}}$$

$$= 128 \text{ ft/sec}$$

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item 5: (1.4, p. 142)

19. Find an equation of the tangent line to the graph of $f(x) = 2/x$ at **(a)** $(1, 2)$; **(b)** $(-1, -2)$;

slope $= m$ at any $x = f'(x)$

$$f(x) = 2x^{-1} \Rightarrow f'(x) = -2x^{-2} = \frac{-2}{x^2}$$

a) $x = 1 \Rightarrow m = f'(1) = \frac{-2}{1^2} = -2$

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = -2(x-1)$$

$$\Rightarrow y = -2x + 4$$

b) $x = -1 \Rightarrow m = f'(-1) = \frac{-2}{(-1)^2} = -2$

$$y - (-2) = -2(x - (-1)) \Rightarrow y + 2 = -2(x+1)$$

$$\Rightarrow y = -2x - 4$$

item 6: (1.5, p. 155)

For each function, find the points on the graph at which the tangent line has slope 1.

81) ~~75~~. $y = 20x - x^2$

$m = \frac{dy}{dx}$

$\frac{dy}{dx} = 20 - 2x$

$20 - 2x = 1 \Rightarrow$

$-2x = -19 \Rightarrow$

$x = \frac{19}{2}$

$y = 20\left(\frac{19}{2}\right) - \left(\frac{19}{2}\right)^2$

$= 190 - \frac{361}{4}$

$= \frac{399}{4}$

$\left(\frac{19}{2}, \frac{399}{4}\right)$

82) ~~76~~. $y = 6x - x^2$

$\frac{dy}{dx} = 6 - 2x$

$6 - 2x = 1 \Rightarrow x = \frac{5}{2}$

$y = 6\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2$

$= 15 - \frac{25}{4}$

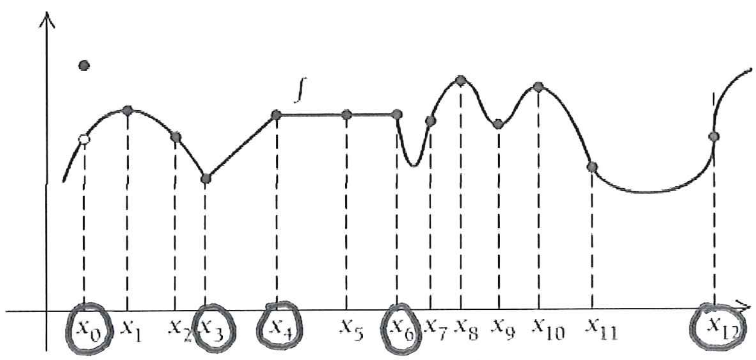
$= \frac{60}{4} - \frac{25}{4} = \frac{35}{4}$

$\left(\frac{5}{2}, \frac{35}{4}\right)$

item 7: (1.4, p. 142)

Find where f(x) is not differentiable

25.



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item 8: (2.3, p. 247)

50. $f(x) = \dfrac{x^2 + x - 2}{2x^2 - 2}$ (Find the asymptotes & id any holes)

$$= \frac{(x+2)(x-1)}{2(x^2-1)} = \frac{(x+2)\cancel{(x-1)}}{2(x+1)\cancel{(x-1)}}$$

$$f(x) = \frac{1x^2 + \dots}{2x^2 + \dots}$$

HA: $y = \frac{1}{2}$

VA: $x = -1$

hole at $x = 1$

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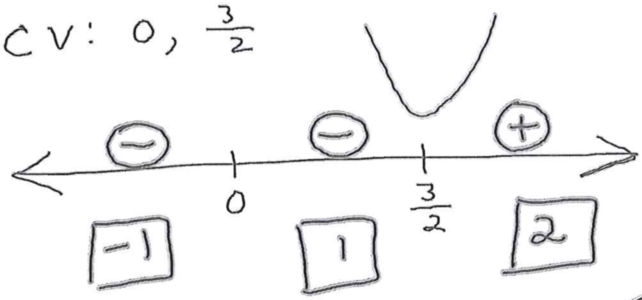
item 11: (2.1, p. 212)

18. $f(x) = x^4 - 2x^3$ (Find Relative Extrema)

$$f'(x) = 4x^3 - 6x^2$$
$$= 2x^2(2x-3) \Rightarrow f'(x) = 0$$
when $x = 0,\ x = \frac{3}{2}$

CV: $0,\ \frac{3}{2}$



Rel. Min at
$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{-27}{16}$

$\left(\frac{3}{2}, \frac{-27}{16}\right)$

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item 12: (2.2, p. ~~228~~)

30. $f(x) = x^3 + 3x + 1$ (Determine Concavity)

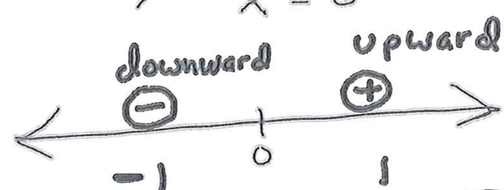
$f'(x) = 3x^2 + 3$

$f''(x) = 6x$

let $f''(x) = 0 \Rightarrow 6x = 0$

$\Rightarrow x = 0$

CV'': 0



down: $(-\infty, 0)$

up: $(0, \infty)$

inf pt. $(0, f(0)) = (0, 1)$

292

item 17: (2.7, p. ~~288~~)

16. $x^2 + \underbrace{2xy}_{\text{product rule}} = 3y^2$ $\quad (2x)y$ $\qquad$ (find $\frac{dy}{dx}$)

$$2x + (2x)\frac{dy}{dx} + y(2) = 6y\frac{dy}{dx} \Rightarrow$$

$$2x\frac{dy}{dx} - 6y\frac{dy}{dx} = -2x - 2y \Rightarrow$$

$$\frac{dy}{dx}(2x - 6y) = -2x - 2y \Rightarrow$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x - 6y} = \frac{-2(x+y)}{2(x-3y)}$$

$$= \frac{-(x+y)}{x-3y} \quad \text{- OR -} \quad = \frac{x+y}{3y-x}$$

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item 18: (2.7, p. ~~289~~)

40. Rate of change of a healing wound. The area of a healing wound is given by

$A = \pi r^2.$

The radius is decreasing at the rate of 1 millimeter per day (−1 mm/day) at the moment when $r = 25$ mm. How fast is the area decreasing at that moment?

$\frac{dr}{dt} = -1$

$\frac{dA}{dt} = ?$

differentiate

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(25\,mm)(-1\,mm/day)$$

$$= -50\pi\ mm^2/day$$

item 23: (3.2, p. $\cancel{332}$ 334)

~~60~~ 66). $g(x) = e^{2x} \cdot \ln x$ (find $g'(x)$)

$$g'(x) = e^{2x} \cdot \frac{1}{x} + (\ln x) \cdot 2e^{2x}$$

$$= e^{2x}\left(\frac{1}{x} + 2\ln x\right)$$

$$= e^{2x}\left(\frac{1}{x} + \frac{2x\ln x}{x}\right)$$

$$= e^{2x}\left(\frac{1+2x\ln x}{x}\right)$$

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item 24: (3.5, p. 368)

12. $f(x) = 12^{7x-4}$ (find $f'(x)$)

$f'(x) = 7(12^{7x-4})\ln 12$

$\frac{d}{dx}(b^u) =$

$u' b^u \ln b$

407

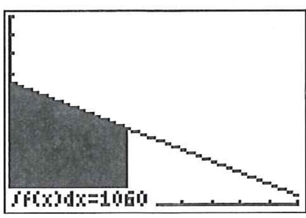
item 25: (4.1, p. ~~398~~)

1. Total cost from marginal cost. Redline Roasting has found that the cost, in dollars per pound, of the coffee it roasts is

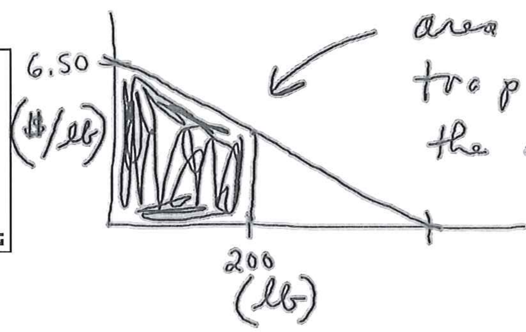
Rate

$$c(x) = -0.012x + 6.50, \quad \text{for } x \leq 300,$$

where x is the number of pounds of coffee roasted. Find the total cost of roasting 200 lb of coffee.



$1060



area of trapezoid is the answer

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4.1, p. 397

item 27 (~~4.2, p. 411~~)

65. Demand from marginal demand. A company finds that the rate at which the quantity of a product that consumers demand changes with respect to price is given by the marginal-demand function

$$D'(x) = -\frac{4000}{x^2},$$

where x is the price per unit, in dollars. Find the demand function if it is known that 1003 units of the product are demanded by consumers when the price is \$4 per unit.

$D(4) = 1003$

$$D(x) = \int D'(x)\,dx = \int 4000x^{-2}\,dx$$

$$= \frac{4000x^{-1}}{-1} + C = \frac{-4000}{x} + C$$

$$\frac{-4000}{4} + C = 1003 \Rightarrow C = 2003$$

$$D(x) = \frac{-4000}{x} + 2003$$

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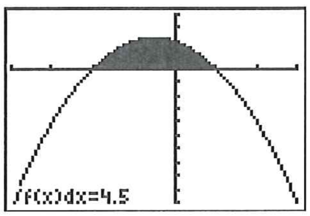
item 29: (4.3, p. 422)

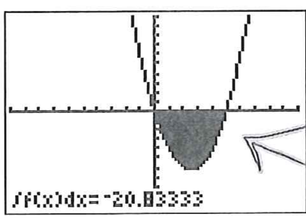
28. $y = 2 - x - x^2$; $[-2, 1]$ (Find area under curve)



item 30: (4.4, p. 434)

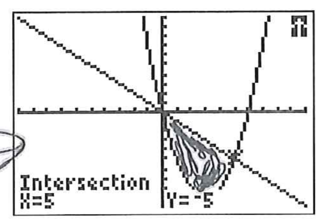
26) $\cancel{22}$. $y = x^2 - 6x,\ y = -x$ (Find area between the given curves)

```
Plot1 Plot2 Plot3
\Y1=X²-6X-(-X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```




```
Ans▸Frac
          -125/6
abs(Ans)
     20.83333333
Ans▸Frac
           125/6
```

$\frac{125}{6}$



same area

item 31: (4.4, p. 435)

39) ~~35~~. $y = 4 - x^2$; $[-2, 2]$ (Find average value of function)

$$\text{avg of } f(x) \text{ on } [a,b] = \frac{\int_a^b f(x)\,dx}{b-a}$$

$$\int_{-2}^{2} (4 - x^2)\,dx = 4x - \frac{x^3}{3}\Big|_{-2}^{2}$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{+8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3}$$

$$= 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3} = \frac{32}{3}$$

$$\text{Avg} = \frac{\frac{32}{3}}{2-(-2)} = \frac{\frac{32}{3}}{4} = \frac{32}{12} = \frac{8}{3}$$

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item 32: (4.5, p. 443)

22. $\int \frac{dx}{1+7x}$ =

let $u = 1+7x$

$\frac{du}{dx} = 7$

$dx = \frac{du}{7}$

$\frac{1}{7}\int \frac{du}{u}$ =

$\frac{1}{7}\ln|u| + C$ =

$\frac{1}{7}\ln|1+7x| + C$

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item 33: (4.5, p. 443)

32. $\int \frac{(\ln x)^2}{x}\,dx =$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\int \frac{u^2}{x} \cdot x\,du =$

$\int u^2\,du = \frac{u^3}{3} + c$

$= \frac{(\ln x)^3}{3} + c$

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