Differentiation Techniques: The Power and Sum-Difference Rules

1.5

OBJECTIVE

- Differentiate using the Short-Cut Rules
- Determine points at which a tangent line has a specified slope.

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Leibniz's Notation:

When y is a function of x, we will also designate the derivative, $f'(x)$, as

$$\frac{dy}{dx},$$

which is read "the derivative of y with respect to x."

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THEOREM 1: The Power Rule

For any real number *n*,

$$\frac{dy}{dx}x^n = n \cdot x^{n-1}$$

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Example 1: Differentiate each of the following:

a) $y = x^5$ b) $y = x$ c) $y = x^{-4}$

a) $\frac{d}{dx}x^5 = 5 \cdot x^{5-1}$ b) $\frac{d}{dx}x = 1 \cdot x^{1-1}$ c) $\frac{d}{dx}x^{-4} = -4x^{-4-1}$

$\frac{d}{dx}x^5 = 5x^4$ $\frac{d}{dx}x = 1$ $\frac{d}{dx}x^{-4} = -4x^{-5}$

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Example 2: Differentiate:

a) $y = \sqrt{x}$ b) $y = x^{0.7}$

a) $\frac{dy}{dx}\sqrt{x} = \frac{dy}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1}$

$\frac{dy}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}}$, *or*

$= \frac{1}{2x^{\frac{1}{2}}}$, *or*

$= \frac{1}{2\sqrt{x}}$

b) $\frac{d}{dx}x^{0.7} = 0.7 \cdot x^{0.7-1}$

$\frac{d}{dx}x^{0.7} = 0.7x^{-0.3}$

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THEOREM 2:

The derivative of a constant times a function is the constant times the derivative of the function. That is,

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}f(x)$$

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Example 3: Find each of the following derivatives:

a) $\frac{d}{dx}7x^4$ b) $\frac{d}{dx}\left(\frac{1}{5x^2}\right)$

a) $\frac{d}{dx}7x^4 = 7 \cdot \frac{d}{dx}x^4$

$= 7 \cdot 4x^{4-1}$

$\frac{d}{dx}7x^4 = 28x^3$

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Example 3 (concluded):

b) $\frac{d}{dx}\left(\frac{1}{5x^2}\right) = \frac{d}{dx}\left(\frac{1}{5}x^{-2}\right)$

$$\frac{d}{dx}\left(\frac{1}{5x^2}\right) = -\frac{2}{5}x^{-3}, \text{ or } = -\frac{2}{5x^3}$$

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Quick Check 2

Differentiate each of the following:

a.) $y = 10x^9,\ y' = 9 \cdot 10x^{9-1} = 90x^8$

b.) $y = \pi x^3,\ y' = 3 \cdot \pi x^{3-1} = 3\pi x^2$

c.) $y = \frac{2}{3x^4} = \frac{2}{3}x^{-4},\ y' = -4\frac{2}{3}x^{-4-1} = -\frac{8}{3}x^{-5} = -\frac{8}{3x^5}$

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THEOREM 3:

The derivative of a constant function is 0. That is,

$$\frac{d}{dx}c = 0$$

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THEOREM 4: The Sum-Difference Rule

Sum: The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

Difference: The derivative of a difference is the difference of the derivatives.

$$\frac{d}{dx}\left[f(x)-g(x)\right] = \frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

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Example 4: Find each of the following derivatives:

a) $\frac{d}{dx}(5x^3 - 7)$ b) $\frac{d}{dx}\left(24x - \sqrt{x} + \frac{5}{x}\right)$

a) $$\frac{d}{dx}(5x^3 - 7) = \frac{d}{dx}(5x^3) - \frac{d}{dx}(7)$$

$$= 5 \cdot \frac{d}{dx}x^3 - 0 = 5 \cdot 3x^{3-1}$$

$$\frac{d}{dx}(5x^3 - 7) = 15x^2$$

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Example 4 (concluded):

b)

$$\frac{d}{dx}\left(24x - \sqrt{x} + \frac{5}{x}\right) = \frac{d}{dx}(24x) - \frac{d}{dx}\sqrt{x} + \frac{d}{dx}\left(\frac{5}{x}\right)$$

$$= 24\cdot\frac{d}{dx}x - \frac{d}{dx}x^{\frac{1}{2}} + 5\cdot\frac{d}{dx}x^{-1}$$

$$= 24\cdot 1x^{1-1} - \frac{1}{2}x^{\frac{1}{2}-1} + 5\cdot -1x^{-1-1}$$

$$= 24 - \frac{1}{2}x^{-\frac{1}{2}} - 5x^{-2}, \quad \text{or} \quad = 24 - \frac{1}{2\sqrt{x}} - \frac{5}{x^2}$$

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Quick Check 3

Differentiate: $y = 3x^5 + 2\sqrt[3]{x} + \frac{1}{3x^2} + \sqrt{5}$

$$\frac{dy}{dx}\left(3x^5 + 2\sqrt[3]{x} + \frac{1}{3x^2} + \sqrt{5}\right) = \frac{dy}{dx}3x^5 + \frac{dy}{dx}2\sqrt[3]{x} + \frac{dy}{dx}\frac{1}{3x^2} + \frac{dy}{dx}\sqrt{5}$$

$$= \frac{dy}{dx}3x^5 + \frac{dy}{dx}2x^{\frac{1}{3}} + \frac{dy}{dx}\frac{1}{3}x^{-2} + \frac{dy}{dx}\sqrt{5} = 5\cdot 3x^{5-1} + \frac{2}{3}x^{\frac{1}{3}-1} - \frac{2}{3}x^{-2-1} + 0$$

$$= 15x^4 + \frac{2}{3}x^{-2/3} - \frac{2}{3}x^{-3} = 15x^4 + \frac{2}{3\sqrt[3]{x^2}} - \frac{2}{3x^3}$$

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Example 5: Find the points on the graph of $f(x) = -x^3 + 6x^2$ at which the tangent line is horizontal.

Recall that the derivative is the slope of the tangent line, and the slope of a horizontal line is 0. Therefore, we wish to find all the points on the graph of *f* where the derivative of *f* equals 0.

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Example 5 (continued):

So, for $f(x) = -x^3 + 6x^2$

$$f'(x) = -3\cdot x^{3-1} + 6\cdot 2x^{2-1}$$

$$f'(x) = -3x^2 + 12x$$

Setting $f'(x)$ equal to 0:

$$-3x^2 + 12x = 0$$

$$-3x(x-4) = 0$$

$$-3x = 0 \quad x - 4 = 0$$

$$x = 0 \quad x = 4$$

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Example 5 (continued):

To find the corresponding *y*-values for these *x*-values, substitute back into $f(x) = -x^3 + 6x^2$.

$$f(0) = -0^3 + 6 \cdot 0^2 \qquad f(4) = -4^3 + 6 \cdot 4^2$$
$$f(0) = 0 \qquad f(4) = 32$$

Thus, the tangent line to the graph of $f(x) = -x^3 + 6x^2$ is horizontal at the points (0, 0) and (4, 32).

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Example 5 (concluded):