Differentiation Techniques: The Product and Quotient Rules

1.6

OBJECTIVE

- Differentiate using the Product and the Quotient Rules.
- Use the Quotient Rule to differentiate the average cost, revenue, and profit functions.

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THEOREM 5: The Product Rule

Let $F(x) = f(x) \cdot g(x)$. Then,

$$F'(x) = \frac{d}{dx}\left[f(x) \cdot g(x)\right]$$

$$F'(x) = f(x) \cdot \frac{d}{dx}g(x) + g(x) \cdot \frac{d}{dx}f(x)$$

That is, the first times the derivative of the second plus the second times the derivative of the first.

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Example : Find $\frac{d}{dx}\left[\begin{array}{cc} x^2+4x & 3x-5 \end{array}\right]$

$$\frac{d}{dx}\left[\begin{array}{cc} x^2+4x & 3x-5 \end{array}\right] =$$

$$x^2+4x\ (3) + \begin{array}{cc} 3x-5 & 2x+4 \end{array} =$$

$$3x^2+12x + \begin{array}{cc} 6x^2+2x-20 & = \end{array}$$

$$9x^2+14x-20$$

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Check

$$\frac{d}{dx}\left[\begin{array}{cc} x^2+4x & 3x-5 \end{array}\right] =$$

$$\frac{d}{dx}\ 3x^3-5x^2+12x^2-20x = 3x^3+7x^2-20x \ =$$

$$9x^2+14x-20$$

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THEOREM 6: The Quotient Rule

If $Q(x) = \dfrac{N(x)}{D(x)}$, then,

$$Q'(x) = \frac{D(x) \cdot N'(x) - N(x) \cdot D'(x)}{D(x)^2}$$

That is, the bottom times the top minus the top times the derivative of the bottom all over the bottom squared.

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Example 2: Differentiate $f(x) = \dfrac{x^2 - 3x}{x - 1}$.

$$f'(x) = \frac{(x-1)(2x-3) - (x^2 - 3x)(1)}{(x-1)^2}$$

$$f'(x) = \frac{2x^2 - 5x + 3 - x^2 + 3x}{(x-1)^2}$$

$$f'(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$$

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DEFINITION:

If *C(x)* is the cost of producing *x* items, then the **average cost** of producing *x* items is $\dfrac{C(x)}{x}$.

If *R(x)* is the revenue from the sale of *x* items, then the **average revenue** from selling *x* items is $\dfrac{R(x)}{x}$.

If *P(x)* is the profit from the sale of *x* items, then the **average profit** from selling *x* items is $\dfrac{P(x)}{x}$.