

Higher Order Derivatives

1.8

OBJECTIVE

- Find derivatives of higher order.
- Given a formula for distance, find velocity and acceleration.

PEARSON

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1.8 Higher Order Derivatives

Higher-Order Derivatives:

Consider the function given by

$$y = f(x) = x^5 - 3x^4 + x.$$

Its derivative f' is given by

$$f'(x) = 5x^4 - 12x^3 + 1.$$

$$f''(x) = 20x^3 - 36x^2.$$

$$f'''(x) = 60x^2 - 72x$$

$$f^{(4)}(x) = 120x - 72$$

$$f^{(5)}(x) = 120$$

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1.8 Higher Order Derivatives

Higher-Order Derivatives (continued):

Leibniz's notation for the second derivative of a function given by $y = f(x)$ is

$$\frac{d^2y}{dx^2}, \text{ or } \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

read "the second derivative of y with respect to x ."

The 2's in this notation are NOT exponents.

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1.8 Higher Order Derivatives

Higher-Order Derivatives (concluded):

If $y = x^5 - 3x^4 + x$, then

$$\frac{dy}{dx} = 5x^4 - 12x^3 + 1, \quad \frac{d^4y}{dx^4} = 120x - 72,$$

$$\frac{d^2y}{dx^2} = 20x^3 - 36x^2, \quad \frac{d^3y}{dx^3} = 120.$$

$$\frac{d^3y}{dx^3} = 60x^2 - 72x,$$

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1.8 Higher Order Derivatives

Example 1: For $y = \frac{1}{x}$, find $\frac{d^2y}{dx^2}$.

$$y = x^{-1}$$

$$\frac{dy}{dx} = -x^{-2}$$

$$\frac{d^2y}{dx^2} = 2x^{-3}, \text{ or } \frac{2}{x^3}$$

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1.8 Higher Order Derivatives

Quick Check 1 Solution Concluded

b.) Find $\frac{d^4}{dx^4} \left[\frac{1}{x} \right]$

$$\begin{aligned} \frac{d^4}{dx^4} \left[\frac{1}{x} \right] &= \frac{d^3}{dx^3} \left[-\frac{1}{x^2} \right] = \frac{d^2}{dx^2} \left[\frac{2}{x^3} \right] = \frac{d}{dx} \left[-\frac{6}{x^4} \right] \\ &= \frac{24}{x^5} \end{aligned}$$

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1.8 Higher Order Derivatives

DEFINITION:

The **velocity** of an object that is $s(t)$ units from a starting point at time t is given by

$$\text{Velocity} = v(t) = s'(t) = \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h}$$

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DEFINITION:

$$\text{Acceleration} = a(t) = v'(t) = s''(t).$$

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1.8 Higher Order Derivatives

Example 3: For $s(t) = 10t^2$ find $v(t)$ and $a(t)$, where s is the distance from the starting point, in miles, and t is in hours. Then, find the distance, velocity, and acceleration when $t = 4$ hr.

$$v(t) = s'(t) = 20t$$

$$a(t) = v'(t) = s''(t) = 20$$

$$s(4) = 10(4)^2 = 160 \text{ mi}$$

$$v(4) = 20(4) = 80 \text{ mi/hr}$$

$$a(4) = 20 \text{ mi/hr}^2$$

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1.8 Higher Order Derivatives

Quick Check 2

A pebble is dropped from a hot-air balloon. Find how far it has fallen, how fast it is falling, and its acceleration after 3.5 seconds. Let $s(t) = 16t^2$, where t is in seconds, and s is in feet.

$$\text{Distance: } s(3.5) = 16(3.5)^2 = 16(12.25) = 196 \text{ feet}$$

$$\text{Velocity: } v(t) = s'(t) = 32t$$

$$v(3.5) = 32(3.5) = 112 \text{ feet/second}$$

$$\text{Acceleration: } a(t) = v'(t) = s''(t) = 32$$

$$a(3.5) = 32 \text{ feet/second}^2$$

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