Maximum-Minimum Problems; Business and Economics Applications

2.5

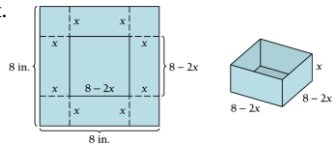
OBJECTIVE

- Solve maximum and minimum problems using calculus.

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 1: From a thin piece of cardboard 8 in. by 8 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

1st make a drawing in which x is the length of each square to be cut.



2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 1 (continued):

2nd write an equation for the volume of the box.

$$\begin{aligned} V &= l \cdot w \cdot h \\ V(x) &= (8-2x)\cdot(8-2x)\cdot x \\ V(x) &= (64-32x+4x^2)\cdot x \\ V(x) &= 4x^3-32x^2+64x \end{aligned}$$

Note that x must be between 0 and 4. So, we need to maximize the volume equation on the interval (0, 4).

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 1 (continued):

$$\begin{aligned} V' &= 12x^2-64x+64 = 0 \\ & 3x^2-16x+16 = 0 \\ & (3x-4)(x-4) = 0 \\ & 3x-4=0 \quad or \quad x-4=0 \end{aligned}$$

$$x=\frac{4}{3} \quad or \quad x=4$$

$\frac{4}{3}$ is the only critical value in (0, 4). So, we can use the second derivative.

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 1 (concluded):

$$V''(x) = 24x - 64$$

$$V''\left(\frac{4}{3}\right) = 24\left(\frac{4}{3}\right) - 64$$

$$V''\left(\frac{4}{3}\right) = -32 < 0$$

Thus, the volume is maximized when the square corners are $\frac{4}{3}$ inches. The maximum volume is

$$V\left(\frac{4}{3}\right) = 4\left(\frac{4}{3}\right)^3 - 32\left(\frac{4}{3}\right)^2 + 64\left(\frac{4}{3}\right)$$

$$V\left(\frac{4}{3}\right) = 37\frac{25}{27} \text{ in}^3$$

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 2: A stereo manufacturer determines that in order to sell x units of a new stereo, the price per unit, in dollars, must be $p(x) = 1000 - x$. The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 2x$.

a) Find the total revenue $R(x)$.
b) Find the total profit $P(x)$.
c) How many units must the company produce and sell in order to maximize profit?
d) What is the maximum profit?
e) What price per unit must be charged in order to make this maximum profit?

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 2 (continued):

a) Revenue = quantity · price

$$R(x) = x \cdot p$$

$$R(x) = x(1000 - x)$$

$$R(x) = 1000x - x^2$$

b) Profit = Total Revenue − Total Cost

$$P(x) = R(x) - C(x)$$

$$P(x) = 1000x - x^2 - (3000 + 20x)$$

$$P(x) = -x^2 + 980x - 3000$$

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 2 (continued):

c) $P'(x) = -2x + 980 = 0$

$$-2x = -980$$

$$x = 490$$

Since there is only one critical value, we can use the second derivative to determine whether or not it yields a maximum or minimum.

$$P''(x) = -2$$

Since $P''(x)$ is negative, $x = 490$ yields a maximum. Thus, profit is maximized when 490 units are bought and sold.

2.5 Maximum-Minimum Problems; Business and Economics Applications

Example 2 (concluded):

d) The maximum profit is given by

$$P(490) = -(490)^2 + 980(490) - 3000$$
$$P(490) = \$237{,}100.$$

Thus, the stereo manufacturer makes a maximum profit of $237,100 when 490 units are bought and sold.

e) The price per unit to achieve this maximum profit is

$$p(490) = 1000 - 490$$
$$p(490) = \$510.$$