

The Derivatives of a^x and $\log_a x$

3.5

OBJECTIVE

- Differentiate functions involving a^x .
- Differentiate functions involving $\log_a x$.

PEARSON

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Slide 3.5-1

3.5 The Derivatives of a^x and $\log_a x$

Problem: (Recall, if $y = e^u$ then $\frac{dy}{dx} = u'e^u$)

Find $\frac{dy}{dx}$ if $y = 2^x$

$$y = 2^x \Leftrightarrow y = e^{\ln 2^x}$$

$$y = e^{x \ln 2} \Rightarrow$$

$$\frac{dy}{dx} = \frac{d}{dx}(x \ln 2) \cdot e^{x \ln 2} = \ln 2 \cdot e^{x \ln 2} \Leftrightarrow$$

$$\frac{dy}{dx} = (\ln 2) \cdot 2^x$$

THEOREM: $\frac{d}{dx} a^x = (\ln a) \cdot a^x$

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3.5 The Derivatives of a^x and $\log_a x$

Differentiate:

a.) $y = 5^x$, $y' = \ln 5 \cdot 5^x$

b.) $f(x) = 4^x$, $f'(x) = \ln 4 \cdot 4^x$

c.) $y = 4.3^x$, $y' = \ln 4.3 \cdot 4.3^x$

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3.5 The Derivatives of a^x and $\log_a x$

In General,

$$\frac{d}{dx} a^u = u' \cdot (\ln a) \cdot a^u$$

Example

$$\frac{d}{dx} 7^{3x^4 - 5x + 16} = (12x^3 - 5) \cdot \ln(7) \cdot 7^{3x^4 - 5x + 16}$$

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Slide 3.5-4

3.5 The Derivatives of a^x and $\log_a x$

Problem: (Recall, if $y = \ln x$ then $\frac{dy}{dx} = \frac{1}{x}$ and $\log_a x = \frac{\ln x}{\ln a}$)

Find $\frac{dy}{dx}$ if $y = \log_{19} x$

$$y = \frac{\ln x}{\ln 19}$$

$$\frac{dy}{dx} = \left(\frac{1}{x}\right) \frac{1}{\ln 19} = \frac{1}{x \ln 19}$$

THEOREM: $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

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Slide 3.5-5

3.5 The Derivatives of a^x and $\log_a x$

Example: Differentiate:

a) $y = \log_8 x$; b) $y = \log x$;

a) $\frac{d}{dx} \log_8 x = \frac{1}{x \ln 8}$

b) $\frac{d}{dx} \log x = \frac{1}{x \ln 10}$

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Slide 3.5-6

3.5 The Derivatives of a^x and $\log_a x$ In General,

$$\frac{d}{dx} \log_a u = \frac{u'}{u \ln a}$$

Example

$$\frac{d}{dx} \left[\log_7 (2x^7 - 5x^3 + 14x + 11) \right] =$$

$$\frac{14x^6 - 15x^2 + 14}{2x^7 - 5x^3 + 14x + 11 \ln 7}$$

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