The Derivatives of a^x and $\log_a x$

OBJECTIVE

- Differentiate functions involving a^x.
- Differentiate functions involving $\log_a x$.

3.5 The Derivatives of a^x and $\log_a x$

Problem: (Recall, if $y = e^u$ then $\frac{dy}{dx} = u'e^u$)

Find $\frac{dy}{dx}$ if $y = 2^x$

$y = 2^x \Leftrightarrow y = e^{\ln 2^x}$

$y = e^{x \ln 2} \Rightarrow$

$\frac{dy}{dx} = \frac{d}{dx}(x \ln 2) \cdot e^{x \ln 2} = \ln 2 \cdot e^{x \ln 2} \Leftrightarrow$

$\frac{dy}{dx} = (\ln 2) \cdot 2^x$

THEOREM: $\frac{d}{dx} a^x = (\ln a) \cdot a^x$

3.5 The Derivatives of a^x and $\log_a x$

Differentiate:

a.) $y = 5^x$, $y' = \ln 5 \ 5^x$

b.) $f(x) = 4^x$, $f'(x) = \ln 4 \ 4^x$

c.) $y = 4.3^x$, $y' = \ln 4.3 \ 4.3^x$

3.5 The Derivatives of a^x and $\log_a x$

In General,

$$\frac{d}{dx} a^u = u' \cdot (\ln a) \cdot a^u$$

Example

$$\frac{d}{dx} 7^{3x^4 - 5x + 16} = 12x^3 - 5 \cdot \ln(7) \cdot 7^{3x^4 - 5x + 16}$$

3.5 The Derivatives of a^x and $\log_a x$

Problem: $\left(\text{Recall, if } y = \ln x \text{ then } \frac{dy}{dx} = \frac{1}{x} \text{ and } \log_a x = \frac{\ln x}{\ln a}\right)$

Find $\frac{dy}{dx}$ if $y = \log_{19} x$

$y = \frac{\ln x}{\ln 19}$

$\frac{dy}{dx} = \left(\frac{1}{x}\right)\frac{1}{\ln 19} = \frac{1}{x \ln 19}$

THEOREM: $\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$

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Example: Differentiate:

a) $y = \log_8 x$; b) $y = \log x$;

a) $\frac{d}{dx}\log_8 x = \frac{1}{x \ln 8}$

b) $\frac{d}{dx}\log x = \frac{1}{x \ln 10}$

3.5 The Derivatives of a^x and $\log_a x$

In General,

$$\frac{d}{dx}\log_a u = \frac{u'}{u \ln a}$$

Example

$$\frac{d}{dx}\left[\log_7 \left(2x^7 - 5x^3 + 14x + 11\right)\right] =$$

$$\frac{14x^6 - 15x^2 + 14}{\left(2x^7 - 5x^3 + 14x + 11\right)\ln 7}$$