# Properties of Definite Integrals


***OBJECTIVE***

- Use the properties of definite integrals to find the area between curves.
- Solve applied problems involving definite integrals.
- Determine the average value of a function.



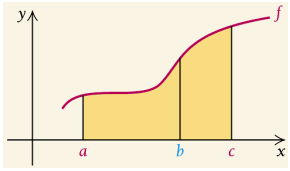


## 4.4 Properties of Definite Integrals

**THEOREM**

For $a < b < c$, $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

For any number $b$ between $a$ and $c$, the integral from $a$ to $c$ is the integral from $a$ to $b$ plus the integral from $b$ to $c$.



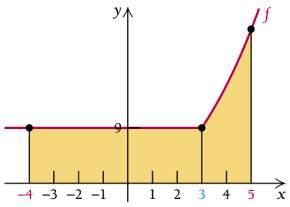





## 4.4 Properties of Definite Integrals

**Example:** Find the area under the graph of $y = f(x)$ from $-4$ to 5, where

$$f(x) = \begin{cases} 9, & \text{for } x < 3, \\ x^2, & \text{for } x \geq 3. \end{cases}$$








## 4.4 Properties of Definite Integrals

**Example (concluded):**

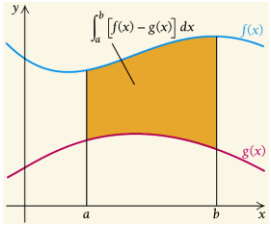
$$\begin{aligned} \int_{-4}^{5} f(x)dx &= \int_{-4}^{3} f(x)dx + \int_{3}^{5} f(x)dx \\ &= \int_{-4}^{3} 9\,dx + \int_{3}^{5} x^2 dx \\ &= 9\,x\,\Big|_{-4}^{3} + \left[\frac{x^3}{3}\right]_{3}^{5} \\ &= 9\,[3-(-4)] + \left(\frac{5^3}{3} - \frac{3^3}{3}\right) \\ &= 95\frac{2}{3} \end{aligned}$$

## 4.4 Properties of Definite Integrals

**THEOREM**

Let $f$ and $g$ be continuous functions and suppose that $f(x) \geq g(x)$ over the interval $[a, b]$. Then the area of the region between the two curves, from $x = a$ to $x = b$, is

$$\int_a^b \left[ f(x) - g(x) \right] dx.$$



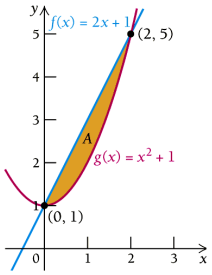




## 4.4 Properties of Definite Integrals

**Example:** Find the area of the region that is bounded by the graphs of $f(x) = 2x + 1$ and $g(x) = x^2 + 1$.

First, look at the graph of these two functions.







## 4.4 Properties of Definite Integrals

**Example (continued):**

Second, find the points of intersection by setting $f(x) = g(x)$ and solving.

$$\begin{aligned} f(x) &= g(x) \\ 2x + 1 &= x^2 + 1 \\ 0 &= x^2 - 2x \\ 0 &= x(x - 2) \\ x = 0 &\quad \text{or} \quad x = 2 \end{aligned}$$



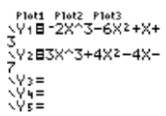

## 4.4 Properties of Definite Integrals

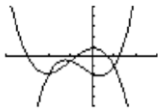
**Example 2 (concluded):**

Lastly, compute the integral. Note that on $[0, 2]$, $f(x)$ is the upper graph.

$$\begin{aligned} \int_0^2 \left[ (2x + 1) - (x^2 + 1) \right] dx &= \int_0^2 (2x - x^2)\, dx \\ &= \left[ x^2 - \frac{x^3}{3} \right]_0^2 \\ &= \left( 2^2 - \frac{2^3}{3} \right) - \left( 0^2 + \frac{0^3}{3} \right) \\ &= 4 - \frac{8}{3} - 0 + 0 = \frac{4}{3} \end{aligned}$$

Find the area between the two curves: $y = -2x^3 - 6x^2 + x + 3$, $y = 3x^3 + 4x^2 - 4x - 7$



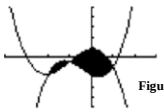


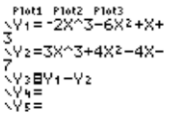
Figure 1

They intersect at -2, -1, and 1





Turn Off Y1 and Y2



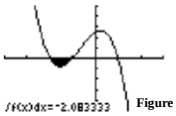
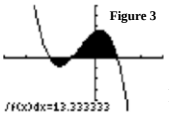




Figure 2

Take absolute value, 2.08333… = 25/12

Figure 3




13.333…= 40/3

The area here is the same as the one in **figure 1**

**Final Answer** is 25/12 + 160/12 = 185/12 or 15 5/12





## 4.4 Properties of Definite Integrals

**DEFINITION:**

Let $f$ be a continuous function over a closed interval $[a, b]$. Its **average value**, $y_{av}$, over $[a, b]$ is given by

$$y_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

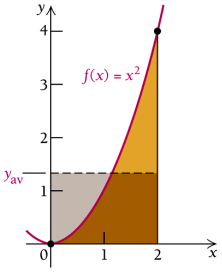




## 4.4 Properties of Definite Integrals

**Example 4:** Find the average value of $f(x) = x^2$ over the interval $[0, 2]$.

$$\frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{1}{2}\left(\frac{2^3}{3} - \frac{0^3}{3}\right) = \frac{1}{2}\cdot\frac{8}{3} = \frac{4}{3} = 1\frac{1}{3}$$



**Example:** Rico's speed, in feet per second, $t$ minutes after entering the freeway is given by

$$v(t) = -\frac{1}{200}t^3 + \frac{3}{20}t^2 - \frac{3}{8}t + 60, \quad t \le 30.$$

From 5 sec after entering the freeway to 25 sec, what was Rico's average speed? How far did he travel over that time interval?